STAT/MA 41600
In-Class Problem Set #10: September 13, 2017
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Problem Set 10 Answers

1a. The probability mass function of X is: $p_X(2) = 1/24$, $p_X(3) = 2/24$, $p_X(4) = 3/24$, $p_X(5) = 4/24$, $p_X(6) = 4/24$, $p_X(7) = 4/24$, $p_X(8) = 3/24$, $p_X(9) = 2/24$, $p_X(10) = 1/24$.
1b. The expected value of X is $\mathbb{E}(X) = (2)(1/24) + (3)(2/24) + (4)(3/24) + (5)(4/24) + (6)(4/24) + (7)(4/24) + (8)(3/24) + (9)(2/24) + (10)(1/24) = 6$.

2a. The probability mass function of X is: $p_X(0) = \binom{5}{0}(1/2)^0(1/2)^5 = 1/32$, $p_X(1) = \binom{5}{1}(1/2)^1(1/2)^4 = 5/32$, $p_X(2) = \binom{5}{2}(1/2)^2(1/2)^3 = 10/32$, $p_X(3) = \binom{5}{3}(1/2)^3(1/2)^2 = 10/32$, $p_X(4) = \binom{5}{4}(1/2)^4(1/2)^1 = 5/32$, $p_X(5) = \binom{5}{5}(1/2)^5(1/2)^0 = 1/32$.
2b. The expected value of X is $\mathbb{E}(X) = \sum_{x=0}^{5}(x)(\binom{5}{x}(1/2)^{5-x}(1/2)^x) = 5/2$.

3a. The probability mass function of X is: $p_X(0) = \binom{4}{0}\binom{48}{5}/\binom{52}{5} = 35673/54145$, $p_X(1) = \binom{4}{1}\binom{48}{4}/\binom{52}{5} = 3243/10829$, $p_X(2) = \binom{4}{2}\binom{48}{3}/\binom{52}{5} = 2162/54145$, $p_X(3) = \binom{4}{3}\binom{48}{2}/\binom{52}{5} = 94/54145$, $p_X(4) = \binom{4}{4}\binom{48}{1}/\binom{52}{5} = 1/54145$, $p_X(5) = \binom{4}{5}\binom{48}{0}/\binom{52}{5} = 0$.

The expected value of X is $\mathbb{E}(X) = \sum_{x=0}^{5}(x)(\binom{4}{x}\binom{48}{5-x}/\binom{52}{5}) = 5/13$.

3b. The probability mass function of X is: $p_X(0) = \binom{5}{0}(4/52)^0(48/52)^5 = 248832/371293$, $p_X(1) = \binom{5}{1}(4/52)^1(48/52)^4 = 103680/371293$, $p_X(2) = \binom{5}{2}(4/52)^2(48/52)^3 = 17280/371293$, $p_X(3) = \binom{5}{3}(4/52)^3(48/52)^2 = 1440/371293$, $p_X(4) = \binom{5}{4}(4/52)^4(48/52)^1 = 60/371293$, $p_X(5) = \binom{5}{5}(4/52)^5(48/52)^0 = 1/371293$.

The expected value of X is $\mathbb{E}(X) = \sum_{x=0}^{5}(x)(\binom{5}{x}(4/52)^x(48/52)^{5-x}) = 5/13$.

4. As in Problem Set 8, we have $P(X = 0) = \frac{\binom{3}{0}\binom{3}{3}}{\binom{6}{3}} = 1/20$ $P(X = 1) = \frac{\binom{3}{1}\binom{3}{2}}{\binom{6}{3}} = 9/20$, $P(X = 2) = \frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} = 9/20$, and $P(X = 3) = \frac{\binom{3}{3}\binom{3}{0}}{\binom{6}{3}} = 1/20$. Therefore, the expected value of X is $\mathbb{E}(X) = (0)(1/20) + (1)(9/20) + (2)(9/20) + (3)(1/20) = 3/2$.