

STAT/MA 41600
In-Class Problem Set #15: September 20, 2017

1. Suppose that X is a Binomial random variable with $n = 6$ and $p = 1/3$. Is X more likely to be even or odd? Why?
2. Let X and Y be independent Binomial random variables with parameters $n = 7$ and $p = 3/4$.
 - 2a. Find $\mathbb{E}(X + Y)$ and $\text{Var}(X + Y)$.
 - 2b. Find $\mathbb{E}(X - Y)$ and $\text{Var}(X - Y)$.
 - 2c. Define $U = X + Y$. Do U and $2X$ have the same distribution, i.e., do they have the same values for their probability mass function? Why or why not?
 - 2d. Find $\mathbb{E}(2X)$ and $\text{Var}(2X)$.
3. Let X and Y be independent Binomial random variables with parameters $n = 5$ and $p = 2/3$.
 - 3a. Find $P(X > Y)$. [Hint: It might be easier to calculate the probability that $X = Y$, and use this to find a faster method of solution.]
 - 3b. What is the variance of $X - Y$?
 - 3c. Calculate the probability that $X - Y$ is 3 or larger, i.e., compute $P(X - Y \geq 3)$.
4. Alice rolls a die until the first occurrence of the value “1” and then she stops. Suppose that it took her exactly 21 rolls to accomplish this, i.e., she rolled 20 times without getting any roll of “1”, and then the 21st roll was a “1”.

Conditional on the information above:

 - 4a. What is the expected number of times that the value “3” appeared?
 - 4b. What is the expected number of times that an even value appeared?
 - 4c. What is the variance of the number of times that an even value appeared?
 - 4d. What is the variance of the number of times that an odd value appeared?