STAT/MA 41600
In-Class Problem Set #24: October 11, 2017
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Problem Set 24 Answers

1a. We have $P(Y > 1/2) = \int_{1/2}^{\infty} 7e^{-7y}\,dy = -e^{-7y}|_{y=1/2}^{\infty} = e^{-7/2} = 0.0302.$

1b. We have $P(0 \leq Y \leq 1/3) = \int_0^{1/3} 7e^{-7y}\,dy = -e^{-7y}|_{y=0}^{1/3} = 1 - e^{-7/3} = 0.9030.$

2a. We have $\int_0^2 (k)(2-x)(3-x)\,dx = \int_0^2 (k)(6-5x+x^2)\,dx = (k)(6x - 5x^2/2 + x^3/3)|_{x=0}^2 = (k)(14/3)$. So we need $k = 3/14$.

2b. We have $P(1 \leq X \leq 2) = \int_1^2 (3/14)(2-x)(3-x)\,dx = \int_1^2 (3/14)(6-5x+x^2)\,dx = (3/14)(6x - 5x^2/2 + x^3/3)|_{x=1}^2 = (3/14)(14/3 - 23/6) = (3/14)(5/6) = 5/28.$

3a. For $a > 0$, we have $F_Y(a) = P(Y \leq a) = P(0 \leq Y \leq a) = \int_0^a 7e^{-7y}\,dy = -e^{-7y}|_{y=0}^a = 1 - e^{-7a}$. For $a \leq 0$, we have $F_Y(a) = 0$.

3b. For $0 < a < 2$, we have $F_X(a) = \int_0^a (k)(2-x)(3-x)\,dx = \int_0^a (3/14)(6-5x+x^2)\,dx = (3/14)(6x - 5x^2/2 + x^3/3)|_{x=0}^a = (3/14)(6a - 5a^2/2 + a^3/3)$. For $a \leq 0$, we have $F_X(a) = 0$. For $a \geq 2$, we have $F_X(a) = 1$.

4a. We have $P(|Y - 1/4| < 1/8) = P(1/8 < Y < 3/8) = \int_{1/8}^{3/8} 7e^{-7y}\,dy = -e^{-7y}|_{y=1/8}^{3/8} = e^{-7/8} - e^{-21/8} = 0.3444.$

4b. We have $1/2 = \int_0^a 7e^{-7y}\,dy = -e^{-7y}|_{y=0}^a = 1 - e^{-7a}$. So we have $e^{-7a} = 1/2$, and thus $-7a = \ln(1/2)$. So we conclude that the median is $a = (1/7)\ln(2) = 0.0990.$