1. Consider a pair of continuous random variables \( X \) and \( Y \) that have the joint probability density function
\[
f_{X,Y}(x,y) = e^{-x-y}, \quad \text{for positive } x \text{ and } y,
\]
and \( f_{X,Y}(x,y) = 0 \) otherwise. Find the probability that \( Y \) is larger than \( 2X + 1 \), i.e., calculate \( P(Y > 2X + 1) \).

2a. For the setup in question 1, find \( P(\max(X, Y) \leq 1) \).
2b. For the setup in question 1, find \( P(\max(X, Y) \leq 2) \).
2c. For the setup in question 1, if \( a > 0 \), find \( P(\max(X, Y) \leq a) \).
2d. Define a new random variable \( Z = \max(X, Y) \). In part c, you essentially calculated the CDF for \( Z \). What is the probability density function for \( Z \)?

3. Suppose that the probability density function for \( X \) is \( f_X(x) = x/9 \) for \( 0 \leq x \leq 3 \), and \( f_X(x) = 2/3 - x/9 \) for \( 3 \leq x \leq 6 \), and \( f_X(x) = 0 \) otherwise.
3a. Calculate \( P(|X - 3| < 1/2) \).
3b. Calculate \( P(|X - 3| > 2) \).

4a. What is the probability density function for the random variable \( X \) defined in question 1?
4b. What is the cumulative distribution function for the random variable \( X \) defined in question 1?
4c. What is the cumulative distribution function for the random variable \( X \) defined in question 3? (You will need to define this in pieces, since the probability density function is defined in pieces.)