

1. Suppose that X and Y have joint probability density function $f_{X,Y}(x, y) = (3/4)(x - y)$ for $0 < y < x < 2$, and $f_{X,Y}(x, y) = 0$ otherwise.
 - 1a. Find $P(Y < 1/2 \mid X = 1)$.
 - 1b. Find $P(Y < 1/2 \mid X < 1)$.

2. Suppose that X and Y have joint probability density function $f_{X,Y}(x, y) = 69e^{-3x-8y}$ for $0 < 5y < x$, and $f_{X,Y}(x, y) = 0$ otherwise.
 - 2a. Find the probability density function of Y .
 - 2b. Find the probability that Y is larger than $1/20$.

- 3a. For the setup in question 2, find the conditional probability density function of X , given $Y = 1/15$. In other words, find $f_{X|Y}(x \mid 1/15)$.
- 3b. For the setup in question 2, find the conditional probability that $X > 1/2$, given that $Y = 1/15$. In other words, find $P(X > 1/2 \mid Y = 1/15)$.

4. Consider a pair of random variables X and Y whose joint probability density function is constant on the triangle with vertices at the points $(-4, 0)$, $(0, 2)$, and $(8, 0)$. What is the conditional probability that X is positive, given that $Y = 1$?