Problem Set 28 Answers

1. We have \( E(X) = \int_0^3 (x)(x/9) \, dx + \int_0^6 (x)(2/3 - x/9) \, dx = x^3/27|_{x=0}^3 + (x^2/3 - x^3/27)|_{x=3}^6 = 1 + (12 - 8) - (3 - 1) = 3. \)

2a. We compute \( E(X) = \int_0^2 \int_0^2 (x)(3/4)(x - y) \, dy \, dx = \int_0^2 (x)(3/4)(xy - y^2/2)|_{y=0}^x \, dx = \int_0^2 (3/4)(x^3/2) \, dx = (3/4)(x^4/8)|_{x=0}^2 = (3/4)(2) = 3/2. \)

2b. We compute \( E(Y) = \int_0^2 \int_0^2 (y)(3/4)(x - y) \, dy \, dx = \int_0^2 (3/4)(xy^2/2 - y^3/3)|_{y=0}^x \, dx = \int_0^2 (3/4)(x^3/6) \, dx = (3/4)(x^4/24)|_{x=0}^2 = (3/4)(16/24) = 1/2. \)

3. We compute that \( E(Y) = \int_0^\infty \int_0^{5y} (69e^{-3x-8y}) \, dx \, dy = \int_0^\infty (y)(-23e^{-3x-8y})|_{x=5y}^\infty \, dy = \int_0^\infty (y)(23e^{-23y}) \, dy = (y)(-e^{-23y})|_{y=0}^\infty - \int_0^\infty (-e^{-23y}) \, dy = (-e^{-23y}/23)|_{y=0}^\infty = 1/23. \)

4a. We have \( E(X) = \int_0^2 \int_{2y-4}^-(-4y+8) (x)(1/12) \, dx \, dy = \int_0^2 (x^2/2)(1/12)|_{x=2y-4}^2 \, dy = \int_0^2 ((-4y+8)^2/2 - (2y-4)^2/2)(1/12) \, dy = \int_0^2 (6y^2 - 24y + 24)(1/12) \, dy = (2y^3 - 12y^2 + 24y)(1/12)|_{y=0}^2 = (16 - 48 + 48)(1/12) = 4/3. \)

4b. We have \( E(Y) = \int_0^2 \int_{2y-4}^-(-4y+8) (y)(1/12) \, dx \, dy = \int_0^2 (xy)(1/12)|_{x=2y-4}^2 \, dy = \int_0^2 ((-4y+8) - (2y-4))(1/12) \, dy = \int_0^2 (y)(-6y + 12)(1/12) \, dy = \int_0^2 (6y^2 + 12y)(1/12) \, dy = (2y^3 + 6y^2)(1/12)|_{y=0} = (16 + 24)(1/12) = 8/12 = 2/3. \)