1a. Let $X$ be the number of minutes after 12 noon when the taxi cab arrives. Then $P(X > 5) = e^{-5/3} = .1889.$

1b. We have $P(X > 7 \mid X > 5) = \frac{P(X > 7 \land X > 5)}{P(X > 5)} = P(X > 7)/P(X > 5) = e^{-7/3}/e^{-5/3} = e^{-2/3} = .5134.$

Alternatively, we could use the memoryless property of exponential random variables, and compute $P(X > 7 \mid X > 5) = P(X > 2) = e^{-2/3} = .5134.$

2a. We have $P(X < Y) = \int_0^\infty \int_x^\infty 21e^{-3x-7y} dy dx = \int_0^\infty -3e^{-3x-7y}|y=x |_y=x dx = \int_0^\infty 3e^{-10x} dx = -3e^{-10x}/10|_x=0 = 3/10.$

2b. We have $P(X < Y) = \int_0^\infty \int_x^\infty (\lambda_1 e^{-\lambda_1 x} - \lambda_1 e^{-\lambda_1 x} + \lambda_2 y) dy dx = \int_0^\infty -\lambda_1 e^{-\lambda_1 x} - \lambda_2 y|y=x |_y=x dx = \int_0^\infty \lambda_1 e^{-(\lambda_1 + \lambda_2)x} dx = -\lambda_1 e^{-(\lambda_1 + \lambda_2)x}/(\lambda_1 + \lambda_2)|_x=0 = \lambda_1 / (\lambda_1 + \lambda_2).$

3. We compute $P(Y < X < 2Y) = \int_0^\infty \int_y^{2y} 21e^{-3x-7y} dy dx = \int_0^\infty -7e^{-3x-7y}|y=x|y=x dy = \int_0^\infty (-7e^{-13y} + 7e^{-10y}) dy = (7e^{-13y}/13 - 7e^{-10y}/10)|_y=0 = (-7/13 + 7/10) = 21/130 = .1615.$

4a. Let $X_1, \ldots, X_5$ denote the respective times, in seconds, until children 1 through 5 take their next breaths. The desired probability is $P(X_1 > 1.2)P(X_2 > 1.2) \cdots P(X_5 > 1.2) = (e^{-1.2/3})^5 = .1353.$

Alternatively, the minimum of five independent exponential random variables is also an exponential random variable, with $\lambda$ equal to the sum of the $\lambda$'s from the five random variables; in this case, $\lambda = 1/3 + \cdots + 1/3 = 5/3.$ So the desired probability is $e^{-(1.2)(5/3)} = .1353.$

4b. As in the “alternative” method for #4a, because the time until the next breath is an exponential random variable with parameter $\lambda = 1/3 + \cdots + 1/3 = 5/3,$ then the expected time until the next child takes a breath is $1/\lambda = 3/5$ seconds.