STAT/MA 41600
In-Class Problem Set #33: November 1, 2017
Solutions by Mark Daniel Ward

Problem Set 33 Answers

1. The time (in seconds) until the arrival of the second car is a Gamma random variable with parameters $r = 2$ and $\lambda = 1/30$. Therefore, the probability that the second car passes within the next 1 minute (i.e., 60 seconds) is $1 - e^{-60/30} \sum_{j=0}^{1} \frac{(60/30)^j}{j!} = 1 - 3e^{-2} = 0.5940$.

2. The total time (in minutes) that he spends waiting is a Gamma random variable with parameters $r = 4$ and $\lambda = 1/3$. So the probability that he spends at least 10 minutes waiting is $e^{-10/3} \sum_{j=0}^{3} \frac{(10/3)^j}{j!} = 1301e^{-10/3}/81 = 0.5730$.

3a. Since X (which is given in hours) is a Gamma random variable with $r = 100$ and $\lambda = 120$, then we get $\text{Var}(X) = r/\lambda^2 = 100/120^2 = 1/144 = 0.0069$.
3b. The variance in question 2 is $r/\lambda^2 = 4/(1/3)^2 = 36$.

4a. We let $X = X_1 + \cdots + X_{10}$, where $X_j = 1$ if the jth group has 3 red bears, and $X_j = 0$ otherwise. So we get $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_{10}) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_{10}) = 10\mathbb{E}(X_1) = 10(10/30)(9/29)(8/28) = 60/203 = 0.2956$.
4b. We have $\mathbb{E}(X^2) = \mathbb{E}((X_1 + \cdots + X_{10})^2) = 10\mathbb{E}(X_1^2) + 90\mathbb{E}(X_1X_2) = 10\mathbb{E}(X_1) + 90\mathbb{E}(X_1X_2) = 10(10/30)(9/29)(8/28) + 90(10/30)(9/29)(8/28)(7/27)(6/26)(5/25) = \frac{864}{2639} = 0.3274$. So we conclude that $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 864/2639 - (60/203)^2 = 128592/535717 = 0.2400$.