

Problem Set 41 Answers

1a. Let X be the time until the next message appears. Then $P(X \geq 40) \leq 26/40 = 13/20$, by the Markov Inequality.

1b. We compute $P(20 \leq X \leq 32) = P(|X - 26| \leq 6) = P(|X - \mu_X| \leq (3/2)(\sigma_X)) \geq 1 - 1/(3/2)^2 = 1 - 4/9 = 5/9$, where the inequality holds by Chebyshev's Inequality.

2. Let X be the number of candies. Chebyshev's Inequality gives $P(515 \leq X \leq 575) = P(|X - 545| \leq 30) = P(|X - \mu_X| \leq (3)(\sigma_X)) \geq 1 - 1/3^2 = 8/9$.

3. We compute:

$$\begin{aligned} P(X \leq Y) &= \sum_{x=1}^{\infty} \sum_{y=x}^{\infty} (2/3)^{x-1} (1/3) (4/5)^{y-1} (1/5) \\ &= \sum_{x=1}^{\infty} (2/3)^{x-1} (1/3) (1/5) \sum_{y=x}^{\infty} (4/5)^{y-1} \\ &= \sum_{x=1}^{\infty} (2/3)^{x-1} (1/3) (1/5) (4/5)^{x-1} / (1 - 4/5) \\ &= \sum_{x=1}^{\infty} (8/15)^{x-1} (1/3) \\ &= (1/3) / (1 - 8/15) \\ &= 5/7 \end{aligned}$$

4. We compute $P(X \leq Y) = \int_0^{\infty} \int_x^{\infty} 120e^{-10x-12y} dy dx = \int_0^{\infty} -10e^{-10x-12y} \Big|_{y=x}^{\infty} dx = \int_0^{\infty} 10e^{-22x} dx = -(10/22)e^{-22x} \Big|_{x=0}^{\infty} = 10/22 = 5/11$.