1. Suppose that $X$ is an Exponential random variable with $\lambda$.

1a. Find the moment generating function $M_X(t)$ of $X$.

1b. Compute $M_X'(0)$. Hint: You should get $1/\lambda$ for your answer, since $M_X'(0) = \mathbb{E}(X)$.

1c. Compute $M_X''(0)$. Hint: You should get $2/\lambda^2$ for your answer, since $M_X''(0) = \mathbb{E}(X^2)$.

(We learned these facts in 1b and 1c on October 27, 2017, in the notes for Problem Set 32.)

2. Same setup as in 1.

2a. Compute $\mathbb{E}(X^3) = M_X'''(0)$. (This would previously have taken 3 integrations by parts!)

2b. Compute $\mathbb{E}(X^4) = M_X''''(0)$. (This would previously have taken 4 integrations by parts!)

2c. Can you find a general formula for $\mathbb{E}(X^n) = M_X^{(n)}(0)$?

3. Suppose that $X$ is a Chi-squared random variable with parameter $k$. Then $X$ has moment generating function $(1 - 2t)^{-k/2}$. (Technical point: this MGF is valid for $t < 1/2$.)

3a. Find $\mathbb{E}(X)$.

3b. Find $\mathbb{E}(X^2)$.

3c. Use your answers above to find $\text{Var}(X)$.

4. Suppose random variable $X$ has probability mass function $P(X = x) = (125/156)(1/5)^x$, for integers $0 \leq x \leq 3$.

   a. Verify that this is a valid probability mass function.

   b. Manually compute the expected value of $X$.

   c. Find the moment generating function $M_X(t)$ of $X$. (If you think for a moment, it is possible to write $M_X(t)$ without using any summation signs or addition symbols.)

   d. Compute $M_X'(0)$. Hint: Your answer should agree with your answer for 4b.