

STAT/MA 41600
Midterm Exam #2: November 20, 2017

Name _____

Purdue student ID (10 digits) _____

1. The testing booklet contains 5 questions, but students only need to answer 4 of the questions. The 4 questions chosen by the student will all be weighted evenly (i.e., each question is worth $1/4$ of the midterm exam grade).
2. Permitted Texas Instruments calculators:
 - BA-35
 - BA II Plus*
 - BA II Plus Professional Edition*
 - TI-30XS MultiView*
 - TI-30Xa
 - TI-30XIIS*
 - TI-30XIIB*
 - TI-30XB MultiView*

*The memory of the calculator should be cleared at the start of the exam.
3. **Circle your final answer in your booklet;** otherwise, no credit may be given.
4. There is no penalty for guessing or partial work.
5. Show all your work in the exam booklet. If the majority of questions are answered correctly, but insufficient work is given, the exam could be considered for academic misconduct.
6. Extra sheets of paper are available from the proctor.

1. Consider a continuous random variable Y that has the probability density function $f_Y(y) = 7e^{-7y}$ for $y > 0$, and $f_Y(y) = 0$ otherwise.

1a. For the random variable Y in question 1, calculate $P(|Y - 1/4| < 1/8)$.

1b. For the random variable Y in question 1, what is the median? In other words, for which value of “ a ” do we have $P(Y \leq a) = 1/2$?

Grade this question? Yes No

2. Consider a Beta random variable X with parameters $\alpha = 2$ and $\beta = 2$.
Find $P(1/4 < X < 3/4)$.

Grade this question? Yes No

3. Suppose that X and Y have joint probability density function $f_{X,Y}(x, y) = 21e^{-3x-4y}$ for $0 < y < x$, and $f_{X,Y}(x, y) = 0$ otherwise.

3a. Find the conditional probability density function of X , given $Y = 2$. In other words, find $f_{X|Y}(x | 2)$.

3b. Find the conditional probability that $X > \frac{5}{2}$, given that $Y = 2$. In other words, find $P(X > \frac{5}{2} | Y = 2)$.

Grade this question? Yes No

4. Define $f(x) = x^3/324$ for $0 < x < 6$ and $f(x) = 0$ otherwise. Suppose X_1, \dots, X_{100} are independent, continuous random variables that each have probability density function $f(x)$.

4a. Find $\mathbb{E}(X_j)$.

4b. Find $\text{Var}(X_j)$.

4c. Find a good estimate for $P(X_1 + \dots + X_{100} < 475)$.

Grade this question? Yes No

5. Consider 10 fish in a bowl: 8 of them are red, and 1 is green, and 1 is blue. Select the fish one at a time, without replacement, until the bowl is empty.

Let $X = 1$ if all of the red fish are selected, before the green fish is selected; and $X = 0$ otherwise.

Let $Y = 1$ if all of the red fish are selected, before the blue fish is selected; and $Y = 0$ otherwise.

Find the covariance of X and Y .

Grade this question? Yes No