

STAT/MA 41600  
Midterm Exam 2 Answers  
Monday, November 20, 2017  
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**1a.** We have  $P(|Y - 1/4| < 1/8) = P(1/8 < Y < 3/8) = \int_{1/8}^{3/8} 7e^{-7y} dy = -e^{-7y}|_{y=1/8}^{3/8} = e^{-7/8} - e^{-21/8} = 0.3444$ .

**1b.** We have  $1/2 = \int_0^a 7e^{-7y} dy = -e^{-7y}|_{y=0}^a = 1 - e^{-7a}$ . So we have  $e^{-7a} = 1/2$ , and thus  $-7a = \ln(1/2)$ . So we conclude that the median is  $a = (1/7) \ln(2) = 0.0990$ .

**2.** We have  $P(1/4 < X < 3/4) = \int_{1/4}^{3/4} \frac{(2+2-1)!}{(2-1)!(2-1)!} x^{2-1} (1-x)^{2-1} dx = \int_{1/4}^{3/4} 6x(1-x) dx = 6(x^2/2 - x^3/3)|_{x=1/4}^{3/4} = 27/32 - 5/32 = 11/16$ .

**3a.** For  $y \leq 0$ , we have  $f_Y(y) = 0$ . For  $y > 0$ , we get  $f_Y(y) = \int_y^\infty 21e^{-3x-4y} dx = -7e^{-3x-4y}|_{x=y}^\infty = 7e^{-7y}$ .

We have  $f_{X|Y}(x | 2) = \frac{f_{X,Y}(x,2)}{f_Y(2)} = \frac{21e^{-3x-8}}{7e^{-14}} = 3e^{-3x+6}$  for  $x > 2$ , and  $f_{X|Y}(x | 2) = 0$  otherwise.

**3b.** We have  $P(X > 5/2 | Y = 2) = \int_{5/2}^\infty f_{X|Y}(x | 2) dx = \int_{5/2}^\infty 3e^{-3x+6} dx = -e^{-3x+6}|_{x=5/2}^\infty = e^{-3/2} = 0.2231$ .

**4a.** We have  $\mathbb{E}(X_j) = \int_0^6 x(x^3/324) dx = 24/5$ .

**4b.** We have  $\mathbb{E}(X_j^2) = \int_0^6 x^2(x^3/324) dx = 24$ . Thus  $\text{Var}(X) = 24 - (24/5)^2 = 24/25$ .

**4c.** We have  $P(X_1 + \dots + X_{100} < 475) = P\left(\frac{X_1 + \dots + X_{100} - 100(24/5)}{\sqrt{100(24/25)}} < \frac{475 - 100(24/5)}{\sqrt{100(24/25)}}\right) \approx P(Z < -0.51) = P(Z > 0.51) = 1 - P(Z \leq 0.51) = 1 - 0.6950 = 0.3050$ .

**5.** The covariance of  $X$  and  $Y$  is  $\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) = (2/10)(1/9) - (1/9)(1/9) = 4/405 = 0.0099$ .

Question 1 was the very same as question 4 in problem set 24 from 2017.

Question 2 was the very same as question 3b in problem set 34 from 2014.

Question 3 was the same as question 3 in problem set 27 from 2017, with only the numbers in the setup changed.

Question 4 was the same as question 6 in problem set 37 from 2014, with only the numbers in the setup changed.

Question 5 was the same as question 2a in problem set 39 from 2015, with only the numbers in the setup changed.