STAT/MA 41600
In-Class Problem Set #5: August 31, 2018
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Problem Set 5 Answers

1. Let A be the event that the student is in probability, and let B be the event that the student is reading a theorem in a math book when you peek over their shoulder. Then we compute $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{(.10)(.20)}{(.05)(.37)+(.13)(.29)+(.10)(.20)+(.72)(0)} = 0.26$.

2a. Let D be the event that Dean selects a purple bear. Let A_1, B_1, C_1 be the events (respectively) that Mary selected a purple bear, or an orange bear, or a different color of bear. In particular, C_1 is the event that Mary was unhappy. We compute $P(C_1 \mid D) = \frac{P(C_1\cap D)}{P(D)} = \frac{P(C_1\cap D)}{P(A_1\cap D)+P(B_1\cap D)+P(C_1\cap D)} = \frac{(4/6)(1/5)}{(1/6)(3/8)+(1/6)(1/8)+(4/6)(1/5)} = 8/13$.

2b. Use the same events as above, and also let A_2, B_2, C_2 be the events (respectively) that Mary selected a purple bear, or an orange bear, or a different color of bear, on her second round. In particular, $C_1 \cap C_2$ is the event that Mary was unhappy both times. We compute $P(C_1\cap C_2 \mid D) = \frac{P(C_1\cap C_2\cap D)}{P(D)}$. In the numerator, we have $P(C_1\cap C_2\cap D) = (4/6)(3/5)(1/4) = 1/10$. In the denominator, we have

$$\begin{aligned}
P(D) &= P(A_1 \cap A_2 \cap D) + P(A_1 \cap B_2 \cap D) + P(A_1 \cap C_2 \cap D) + P(B_1 \cap A_2 \cap D) + P(B_1 \cap B_2 \cap D) \\
&\quad + P(B_1 \cap C_2 \cap D) + P(C_1 \cap A_2 \cap D) + P(C_1 \cap B_2 \cap D) + P(C_1 \cap C_2 \cap D) \\
&= (1/6)(3/8)(5/10) + (1/6)(1/8)(3/10) + (1/6)(4/8)(3/7) + (1/6)(1/8)(3/10) + (1/6)(3/8)(1/10) \\
&\quad + (1/6)(4/8)(1/7) + (4/6)(1/5)(3/7) + (4/6)(1/5)(1/7) + (4/6)(3/5)(1/4) \\
&\quad = 23/84 = 0.2738.
\end{aligned}$$

So the desired probability is $P(C_1 \cap C_2 \mid D) = \frac{P(C_1\cap C_2\cap D)}{P(D)} = \frac{1/10}{23/84} = 42/115 = 0.3652$.

3a. Let A denote the event that Rafael has not removed any cards from the deck. Let B denote the event that Susan gets an Ace. Then $P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{(40/52)(4/52)}{(40/52)(4/52)+(12/52)(4/40)} = 100/139 = 0.7194$.

3b. Let A_1, A_2, A_3 denote the event that Rafael has not removed any cards from the deck on the first, second, and third draws, respectively. Let B denote the event that Susan gets an Ace.

When computing $P(B)$, we note that only two things matter, namely, either Rafael avoids removing any cards during the long weekend, which happens with probability $(40/52)^3$, and then Susan has 52 cards to draw from; or Rafael removes cards at least once, which happens with probability $1 - (40/52)^3$, and then Susan has 40 cards to draw from. We get $P(A_1\cap A_2\cap A_3 \mid B) = \frac{P(A_1\cap A_2\cap A_3\cap B)}{P(B)} = \frac{(40/52)^3(4/52)}{(40/52)^3(4/52)+(1-(40/52)^3)(4/40)} = 10000/25561 = 0.3912$.

4. Let A be the event that Alice gets no heads. Let B_n be the event that Bob rolls n times. Then we compute

$$P(A) = \sum_{n=1}^{\infty} P(A\cap B_n) = \sum_{n=1}^{\infty} P(A \mid B_n)P(B_n) = \sum_{n=1}^{\infty}(1/2)^n(5/6)^{n-1}(1/6) = (1/2)(1/6)\sum_{n=1}^{\infty}(5/12)^{n-1}.$$

So we conclude that $P(A) = (1/2)(1/6)/(1 - 5/12) = 1/7$.