STAT/MA 41600  
In-Class Problem Set #17: September 24, 2018  
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**Problem Set 17 Answers**

1a. The number of interviews is a Negative Binomial random variable with \( r = 3 \) and \( p = 0.53 \), so the expected value is \( r/p = 5.6604 \).

1b. The variance is \( qr/p^2 = 5.0196 \).

1c. The probability we conduct strictly fewer than 6 interviews is \[ \left( \binom{3 - 1}{3 - 1} \right) (0.53)^3(0.47)^{3-3} + \left( \binom{4 - 1}{3 - 1} \right) (0.53)^4(0.47)^{4-3} + \left( \binom{5 - 1}{3 - 1} \right) (0.53)^5(0.47)^{5-3} = 0.5561, \]
so the probability that we conduct 6 or more interviews is \( 1 - 0.5561 = 0.4439 \).

2a. Geometric random variables have the memoryless property, so we get \( P(X > 8 \mid X > 3) = P(X > 5) = q^5 \).

2b. This is the complement of the previous problem; therefore, we have \( P(X \leq 8 \mid X > 3) = 1 - P(X > 8 \mid X > 3) = 1 - q^5 \).

3. We calculate

\[
P(X > Y) = P(X > Y = 0) + P(X > Y = 1) + P(X > Y = 2)
+ P(X > Y = 3) + P(X > Y = 4)
\]

\[
= \binom{4}{0} \left( \frac{1}{3} \right)^0 \left( \frac{2}{3} \right)^2 \binom{4}{1} \left( \frac{1}{3} \right)^1 \left( \frac{2}{3} \right)^3 \binom{4}{2} \left( \frac{1}{3} \right)^2 \left( \frac{2}{3} \right)^4 \\
+ \binom{4}{3} \left( \frac{1}{3} \right)^3 \left( \frac{2}{3} \right)^1 \binom{4}{4} \left( \frac{1}{3} \right)^4 \left( \frac{2}{3} \right)^4 \\
= \binom{4}{0} \left( \frac{2}{9} \right)^0 \left( \frac{2}{3} \right)^4 + \binom{4}{1} \left( \frac{2}{9} \right)^1 \left( \frac{2}{3} \right)^3 + \binom{4}{2} \left( \frac{2}{9} \right)^2 \left( \frac{2}{3} \right)^2 \\
+ \binom{4}{3} \left( \frac{2}{9} \right)^3 \left( \frac{2}{3} \right)^1 + \binom{4}{4} \left( \frac{2}{9} \right)^4 \left( \frac{2}{3} \right)^0 \\
= \sum_{j=0}^{4} \binom{4}{j} \left( \frac{2}{9} \right)^j \left( \frac{2}{3} \right)^{4-j} \\
= \left( \frac{2}{9} + \frac{2}{3} \right)^4 \\
= \left( \frac{8}{9} \right)^4 \\
= 0.6243
\]

4. Method #1: Let \( X_i = 1 \) if the \( i \)th red bear gets selected, and \( X_i = 0 \) otherwise. Then we have \( X = X_1 + \cdots + X_5 \), so \( E(X) = E(X_1) + \cdots + E(X_5) = 3/9 + \cdots + 3/9 = 5/3 \). We compute \( E(X^2) = E((X_1 + \cdots + X_5)^2) = 5E(X_1) + 20E(X_1X_2) = 5(3/9) + 20(3/9)(2/8) = 10/3 \), so we conclude that \( \text{Var}(X) = E(X^2) - (E(X))^2 = 10/3 - (5/3)^2 = 5/9 \).

Method #2: Let \( X_i = 1 \) if the \( i \)th red bear gets selected, and \( X_i = 0 \) otherwise. Then we have \( X = X_1 + X_2 + X_3 \), so \( E(X) = E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3) = 5/9 + 5/9 + 5/9 = 5/3 \). We compute \( E(X^2) = E((X_1 + X_2 + X_3)^2) = 3E(X_1) + 6E(X_1X_2) = 3(5/9) + 6(5/9)(4/8) = 10/3 \), so we conclude that \( \text{Var}(X) = E(X^2) - (E(X))^2 = 10/3 - (5/3)^2 = 5/9 \).