STAT/MA 41600
In-Class Problem Set #17: September 24, 2018
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Problem Set 17 Answers

1a. The number of interviews is a Negative Binomial random variable with $r = 3$ and $p = .53$, so the expected value is $r/p = 5.6604$.
1b. The variance is $qr/p^2 = 5.0196$.
1c. The probability we conduct strictly fewer than 6 interviews is

$$\binom{3-1}{3-1}(.53)^3(.47)^{3-3} + \binom{4-1}{3-1}(.53)^3(.47)^{4-3} + \binom{5-1}{3-1}(.53)^3(.47)^{5-3} = 0.5561,$$

so the probability that we conduct 6 or more interviews is $1 - 0.5561 = 0.4439$.
2a. Geometric random variables have the memoryless property, so we get $P(X > 8 \mid X > 3) = P(X > 5) = q^5$.
2b. This is the complement of the previous problem; therefore, we have $P(X \leq 8 \mid X > 3) = 1 - P(X > 8 \mid X > 3) = 1 - q^5$.
3. We calculate

$$\begin{aligned}
P(X > Y) &= P(X > Y = 0) + P(X > Y = 1) + P(X > Y = 2) \\
&\qquad + P(X > Y = 3) + P(X > Y = 4) \\
&= \binom{4}{0}(1/3)^0(2/3)^4(2/3)^0 + \binom{4}{1}(1/3)^1(2/3)^3(2/3)^1 + \binom{4}{2}(1/3)^2(2/3)^2(2/3)^2 \\
&\qquad + \binom{4}{3}(1/3)^3(2/3)^1(2/3)^3 + \binom{4}{4}(1/3)^4(2/3)^0(2/3)^4 \\
&= \binom{4}{0}(2/9)^0(2/3)^4 + \binom{4}{1}(2/9)^1(2/3)^3 + \binom{4}{2}(2/9)^2(2/3)^2 \\
&\qquad + \binom{4}{3}(2/9)^3(2/3)^1 + \binom{4}{4}(2/9)^4(2/3)^0 \\
&= \sum_{j=0}^{4}\binom{4}{j}(2/9)^j(2/3)^{4-j} \\
&= (2/9 + 2/3)^4 \\
&= (8/9)^4 \\
&= 0.6243
\end{aligned}$$

4. Method #1: Let $X_i = 1$ if the ith bear selected is red, and $X_i = 0$ otherwise. Then we have $X = X_1 + \cdots + X_5$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_5) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_5) = 3/9 + \cdots + 3/9 = 5/3$. We compute $\mathbb{E}(X^2) = \mathbb{E}((X_1 + \cdots + X_5)^2) = 5\mathbb{E}(X_1) + 20\mathbb{E}(X_1X_2) = 5(3/9) + 20(3/9)(2/8) = 10/3$, so we conclude that $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 10/3 - (5/3)^2 = 5/9$.

Method #2: Let $X_i = 1$ if the ith red bear gets selected, and $X_i = 0$ otherwise. Then we have $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 5/9 + 5/9 + 5/9 = 5/3$. We compute $\mathbb{E}(X^2) = \mathbb{E}((X_1 + X_2 + X_3)^2) = 3\mathbb{E}(X_1) + 6\mathbb{E}(X_1X_2) = 3(5/9) + 6(5/9)(4/8) = 10/3$, so we conclude that $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 10/3 - (5/3)^2 = 5/9$.