STAT/MA 41600
In-Class Problem Set #18: September 26, 2018
Solutions by Mark Daniel Ward

Problem Set 18 Answers

1. The sum is $\sum_{x=2}^{\infty} \frac{e^{-5}5^x}{x!} = \sum_{x=0}^{\infty} \frac{e^{-5}5^x}{x!} - \frac{e^{-5}5^0}{0!} - \frac{e^{-5}5^1}{1!} = 1 - e^{-5}(1+5) = 1 - 6e^{-5} = 0.9596$.

2a. The sum of independent Poisson random variables is also a Poisson random variable, whose mean is equal to the sum of the means. In this case, we see that X is a Poisson random variable with mean $(12)(0.5) = 6$.

2b. Since the X_j's are independent, we can add their variances, so that the variance of X is $(12)(0.5) = 6$.

2c. We compute $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = e^{-6}6^0/0! + e^{-6}6^1/1! + e^{-6}6^2/2! + e^{-6}6^3/3! = 61e^{-6} = 0.1512$.

3a. We let X denote the number of errors in such a book chapter. Then we compute $P(X \leq 4) = \sum_{x=0}^{4} P(X = x) = \sum_{x=0}^{4} e^{-2.8}2.8^x/x! = 0.8477$.

3b. We compute $P(X \leq 3 \mid P \leq 5) = \frac{P(X \leq 3 \ \& \ X \leq 5)}{P(X \leq 5)} = \frac{P(X \leq 3)}{P(X \leq 5)} = \frac{\sum_{x=0}^{3} e^{-2.8}2.8^x/x!}{\sum_{x=0}^{5} e^{-2.8}2.8^x/x!} = 0.6919/0.9349 = 0.74$.

4a. The number of defects is a Binomial random variable with $n = 3{,}000{,}000$ and $p = 1/1{,}000{,}000$, so the exact expression for the probability that there are 4 or fewer defects is

$$\sum_{0}^{4} \binom{3{,}000{,}000}{x} \left(\frac{1}{1{,}000{,}000}\right)^x \left(\frac{999{,}999}{1{,}000{,}000}\right)^{3{,}000{,}000-x}.$$

4b. The distribution of the number of defects is approximately Poisson with $\lambda = np = 3$. So the approximation to the probability above is $\sum_{x=0}^{4} e^{-3}3^x/x! = e^{-3}(1 + 3 + 9/2 + 27/6 + 81/24) = e^{-3}131/8 = 0.8153$.