STAT/MA 41600
In-Class Problem Set #19: September 28, 2018
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Problem Set 19 Answers

1a. The number of Wookies that escape is a Hypergeometric random variable with $n = 5$, $N = 20$, and $M = 13$. The probability that only one Wookie escapes is $\binom{13}{1}\binom{7}{4}/\binom{20}{5} = 455/15504 = 0.02935$.
1b. The probability that at least 3 Wookies escape is $\binom{13}{3}\binom{7}{2}/\binom{20}{5} + \binom{13}{4}\binom{7}{1}/\binom{20}{5} + \binom{13}{5}\binom{7}{0}/\binom{20}{5} = 1001/2584 + 5005/15504 + 429/5168 = 6149/7752 = 0.7932$.

2a. The number of attempted languages that work is a Hypergeometric random variable with $n = 10$, $N = 6{,}000{,}000$, and $M = 1{,}000{,}000$. The exact probability that 0 of his 10 attempted languages will work is $\binom{1{,}000{,}000}{0}\binom{5{,}000{,}000}{10}/\binom{6{,}000{,}000}{10}$.
2b. The exact probability that 3 of his 10 languages will work is $\binom{1{,}000{,}000}{3}\binom{5{,}000{,}000}{7}/\binom{6{,}000{,}000}{10}$.
2c. The Binomial approximation to **2a** is $\binom{10}{0}(1/6)^0(5/6)^{10} = 0.1615$. The Binomial approximation to **2b** is $\binom{10}{3}(1/6)^3(5/6)^7 = 0.1550$.

3a. The random variable X, i.e., the number of treated Ewoks who were actually burned, is a Hypergeometric random variable with $n = 10$, $N = 15$, and $M = 6$. So we have $\mathbb{E}(X) = nM/N = (10)(6)/15 = 4$ and $\text{Var}(X) = (n)(M/N)(1 - M/N)(N - n)/(N - 1) = (10)(6/15)(1 - 6/15)(15 - 10)/(15 - 1) = 6/7$.
3b. The probability that X is an odd number is $P(X = 1) + P(X = 3) + P(X = 5) + P(X = 7) + P(X = 9) = \binom{6}{1}\binom{9}{9}/\binom{15}{10} + \binom{6}{3}\binom{9}{7}/\binom{15}{10} + \binom{6}{5}\binom{9}{5}/\binom{15}{10} + \binom{6}{7}\binom{9}{3}/\binom{15}{10} + \binom{6}{9}\binom{9}{1}/\binom{15}{10} = 2/1001 + 240/1001 + 36/143 + 0 + 0 = 38/77 = 0.4935$.

4a. The number of prepared young Jedis in the test is a Hypergeometric random variable with $n = 15$, $N = 30$, and $M = 10$. So the probability that 8 out of the 10 in the test are prepared is $\binom{10}{8}\binom{20}{7}/\binom{30}{15} = 15/667 = 0.0225$
4b. The probability that at least 8 out of the 10 in the test are prepared is $\binom{10}{8}\binom{20}{7}/\binom{30}{15} + \binom{10}{9}\binom{20}{6}/\binom{30}{15} + \binom{10}{10}\binom{20}{5}/\binom{30}{15} = 15/667 + 5/2001 + 1/10005 = 251/10005 = 0.0251$.