1. Suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable $X$, and the time until the reply is denoted by $Y$. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of $X$ and $Y$ is

$$f_{X,Y}(x, y) = \frac{1}{750}e^{-\left(x/150 + y/30\right)}$$

for $y > x > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.

Calculate $P(Y > 50)$.

Hint: Draw the region in the plane where both conditions are satisfied, i.e., $y > x > 0$ and also (simultaneously) $y > 50$. This is the region over which you need to integrate $f_{X,Y}(x, y)$.

2. Consider the joint probability density function from question 1. What is the probability density function of $X$?

3. Suppose that $X$ and $Y$ have a constant joint probability density function on the triangle with vertices at $(0, 0)$, $(0, 6)$, $(10, 0)$, so that $f_{X,Y}(x, y) = 1/30$ for values $(x, y)$ in this triangle, and $f_{X,Y}(x, y) = 0$ otherwise.

What is the probability density function of $X$?

4. Consider the $X$ and $Y$ defined in question 3. Calculate $P(Y < 1)$. 