

STAT/MA 41600  
In-Class Problem Set #25: October 12, 2018

1. Suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable  $X$ , and the time until the reply is denoted by  $Y$ . For this reason, we always have  $Y > X$ .

Suppose that the joint probability density function of  $X$  and  $Y$  is

$$f_{X,Y}(x, y) = \frac{1}{750} e^{-(x/150+y/30)}$$

for  $y > x > 0$ , and  $f_{X,Y}(x, y) = 0$  otherwise.

Calculate  $P(Y > 50)$ .

Hint: Draw the region in the plane where both conditions are satisfied, i.e.,  $y > x > 0$  and also (simultaneously)  $y > 50$ . This is the region over which you need to integrate  $f_{X,Y}(x, y)$ .

2. Consider the joint probability density function from question 1. What is the probability density function of  $X$ ?

3. Suppose that  $X$  and  $Y$  have a constant joint probability density function on the triangle with vertices at  $(0, 0)$ ,  $(0, 6)$ ,  $(10, 0)$ , so that  $f_{X,Y}(x, y) = 1/30$  for values  $(x, y)$  in this triangle, and  $f_{X,Y}(x, y) = 0$  otherwise.

What is the probability density function of  $X$ ?

4. Consider the  $X$  and  $Y$  defined in question 3. Calculate  $P(Y < 1)$ .