STAT/MA 41600
In-Class Problem Set #25: October 12, 2018
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Problem Set 25 Answers

1. We calculate $P(Y > 50) = \int_{50}^{\infty}\int_0^y \frac{1}{750}e^{-(x/150+y/30)}\,dx\,dy = \frac{1}{750}\int_{50}^{\infty} e^{-y/30}\int_0^y e^{-x/150}\,dx\,dy = \frac{1}{750}\int_{50}^{\infty} e^{-y/30}(-150)(e^{-y/150}-1)\,dy = -\frac{1}{5}\int_{50}^{\infty}(e^{-y/25}-e^{-y/30})\,dy = -\frac{1}{5}(-25e^{-y/25}+30e^{-y/30})\big|_{y=50}^{\infty} = 6e^{-5/3} - 5e^{-2}$.

2. For $x \leq 0$, we have $f_X(x) = 0$. For $x > 0$, we compute $f_X(x) = \int_x^{\infty} \frac{1}{750}e^{-(x/150+y/30)}\,dy = \frac{1}{750}e^{-x/150}\int_x^{\infty} e^{-y/30}\,dy = -\frac{1}{25}e^{-x/150}e^{-y/30}\big|_{y=x}^{\infty} = \frac{1}{25}e^{-x/150}e^{-x/30} = \frac{1}{25}e^{-x/25}$.

3. For $x < 0$ and for $x > 10$, we have $f_X(x) = 0$. For $0 \leq x \leq 10$, we compute $f_X(x) = \int_0^{-(3/5)x+6} \frac{1}{30}\,dy = \frac{1}{30}y\big|_0^{-(3/5)x+6} = \frac{1}{30}(-(3/5)x+6) = \frac{1}{150}(30-3x)$

4. We have $P(Y < 1) = \int_0^1\int_0^{-(5/3)y+10} \frac{1}{30}\,dx\,dy = \int_0^1 \frac{1}{30}x\big|_{x=0}^{-(5/3)y+10}\,dy = \int_0^1 \frac{1}{30}(-(5/3)y+10)\,dy = \frac{1}{30}(-(5/3)y^2/2+10y)\big|_{y=0}^1 = \frac{1}{30}(-(5/3)/2+10) = 11/36$.

An alternative approach, which works *only* since the joint probability density function is constant, is to compute the area of the region where $Y < 1$, divided by the area of the entire triangle. The area of the region where $Y < 1$ (inside the triangle) is $(1)(50/6)+(1)(10/6)(1/2) = 50/6+5/6 = 55/6$. The area of the entire triangle is 30. So the desired probability is $(55/6)/30 = 11/36$.

Caution: We only use this alternative approach when the joint distribution function is constant.