STAT/MA 41600
In-Class Problem Set #28: October 19, 2018
Solutions by Mark Daniel Ward

Problem Set 28 Answers

1. We have $\mathbb{E}(X) = \int_0^\infty \int_x^\infty (x)(\frac{1}{750}e^{-(x/150+y/30)})\,dy\,dx = \int_0^\infty (x)(-\frac{1}{25}e^{-(x/150+y/30)})|_{y=x}^\infty\,dx = \int_0^\infty (x)(\frac{1}{25}e^{-x/25})\,dx$. Then we use integration by parts, with $u = x$ and $dv = \frac{1}{25}e^{-x/25}\,dx$. We get $\mathbb{E}(X) = (x)(-e^{-x/25})|_{x=0}^\infty - \int_0^\infty (-e^{-x/25})\,dx = 0 - 25e^{-x/25}|_{x=0}^\infty = 25$.

Alternatively, we could use the probability density function of X, which we derived in Problem Set 25, question 2. Then we get $\mathbb{E}(X) = \int_0^\infty (x)(\frac{1}{25}e^{-x/25})\,dx$, and we use integration by parts (exactly as in the second line of the first paragraph, above) to conclude that $\mathbb{E}(X) = 25$.

2. We compute as follows $\mathbb{E}(X) = \int_0^{10}\int_0^{-3x/5+6}(x)(1/30)\,dy\,dx = \int_0^{10}(x)(1/30)(y)|_{y=0}^{-3x/5+6}\,dx = \int_0^{10}(x)(1/30)(-3x/5+6)\,dx = \int_0^{10}(1/30)(-3x^2/5+6x)\,dx = (1/30)(-x^3/5+3x^2)|_{x=0}^{10} = 10/3$.

Alternative, we could find the probability density function of X, $f_X(x) = \int_0^{-3x/5+6} 1/30\,dy = (1/30)(-3x/5+6)$ for $x > 0$, and $f_X(x) = 0$ otherwise. Then we have $\mathbb{E}(X) = \int_0^{10}(x)(1/30)(-3x/5+6)\,dx = \int_0^{10}(1/30)(-3x^2/5+6x)\,dx = (1/30)(-x^3/5+3x^2)|_{x=0}^{10} = (1/30)(-200+300) = 10/3$.

3. We know $k = 1/36$, from P.S. 26, question 4, so $\mathbb{E}(Y) = \int_0^4\int_0^3 (y)(1/36)(3-x)(4-y)\,dx\,dy = \int_0^4 (y)(1/36)(3x - x^2/2)|_{x=0}^3(4-y)\,dy = \int_0^4 (y)(1/8)(4-y)\,dy = \int_0^4 (1/8)(4y-y^2)\,dy = (1/8)(2y^2 - y^3/3)|_{y=0}^4 = 4/3$.

An alternative method is to use the results of Problem Set, question 4d, namely, that $f_Y(y) = (1/8)(4-y)$ for $0 < y < 4$, and $f_Y(y) = 0$ otherwise. Using this probability density function of Y, we get $\mathbb{E}(Y) = \int_0^4 (y)(1/8)(4-y)\,dy = \int_0^4 (1/8)(4y-y^2)\,dy = (1/8)(2y^2 - y^3/3)|_{y=0}^4 = 4/3$.

4a. We compute $1 = \int_0^2\int_0^2 kx^2y^2\,dy\,dx = \int_0^2 kx^2y^3/3|_{y=0}^2\,dx = \int_0^2 kx^2(8/3)\,dx = k(x^3/3)|_{x=0}^2(8/3) = k(8/3)(8/3) = (k)(64/9)$, so $k = 9/64$. Now we compute $\mathbb{E}(X) = \int_0^2\int_0^2 (x)(9/64)x^2y^2\,dy\,dx = \int_0^2 (x)(9/64)(x^2)(y^3/3)|_{y=0}^2\,dx = \int_0^2 (x)(9/64)(x^2)(8/3)\,dx = \int_0^2 (9/64)(x^3)(8/3)\,dx = 3/2$.

4b. We compute $f_X(x) = \int_0^2 (9/64)(x^2)(y^2)\,dy = (9/64)(x^2)(y^3/3)|_{y=0}^2 = (9/64)(x^2)(8/3) = (9/64)(x^2)(8/3) = (3/8)(x^2)$ for $0 < x < 2$, and $f_X(x) = 0$ otherwise.

4c. Using the pdf of **4b**, we get $\mathbb{E}(X) = \int_0^2 (x)(3/8)(x^2)\,dx = \int_0^2 (3/8)(x^3)\,dx = (3/8)(x^4/4)|_{x=0}^2 = 3/2$.