STAT/MA 41600
In-Class Problem Set #31: October 24, 2018
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Problem Set 31 Answers

1. The CDF is
\[
F_X(x) = \begin{cases} 
0 & \text{if } x < -3 \\
\frac{x}{6} + \frac{1}{2} & \text{if } -3 \leq x \leq 3 \\
1 & \text{if } x > 3 
\end{cases}
\]

2. We compute as follows
\[
P(Y > X) = \int_{-5}^{5} \int_{-\infty}^{\infty} \left(\frac{1}{25}\right)^2 e^{-2y} dx dy = \int_{0}^{5} \left(\frac{1}{5}\right)e^{-2x} dx = \left(\frac{1}{5}\right) \left(1 - e^{-10}\right).
\]

3. For 0 \leq a \leq 20, we have
\[
P(V \geq a) = P(X \geq a \& Y \geq a \& Z \geq a) = P(X \geq a)P(Y \geq a)P(Z \geq a) = (1 - a/20)^3,
\]
and it follows that the density of V is
\[
f_V(v) = \left(\frac{3}{20}\right)(1 - v/20)^2,
\]
and the expected value of V is
\[
E(V) = \int_{20}^{\infty} \left(\frac{3}{20}\right)(v - v^2/10 + v^3/400) dv = \left(\frac{3}{20}\right)(v^2/2 - v^3/30 + v^4/1600)|_{v=0} = 5.
\]

4. We compute
\[
E(Y) = \int_{0}^{5} \int_{y-5}^{y} \left(\frac{1}{25}\right)^2 (10 - 2y) dx dy = \int_{0}^{5} \left(\frac{1}{25}\right)(10y - 5y^2) dy = \int_{0}^{5} \left(\frac{1}{25}\right)(2y^3 - 30y^2 + 10y) dy = \left(\frac{1}{25}\right)(10y^4/3 - 2y^3 + 10y^2/3)|_{y=0} = 5/3.
\]
Alternatively, the density of Y is
\[
f_Y(y) = \frac{1}{25}(10 - 2y) dx = \left(\frac{1}{25}\right)(10 - y - 5) = (1/25)(10 - 2y) for 0 \leq y \leq 5, and f_Y(y) = 0 otherwise. Then we get
\[
E(Y) = \int_{0}^{5} \left(\frac{1}{25}\right)(10 - 2y) dy, and we proceed as we did in the paragraph above.