STAT/MA 41600
In-Class Problem Set #31: October 24, 2018
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Problem Set 31 Answers

1. The CDF is

$$F_X(x) = \begin{cases} 0 & \text{if } x < -3 \\ x/6 + 1/2 & \text{if } -3 \le x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

2. We compute as follows $P(Y > X) = \int_0^5 \int_x^\infty (1/5)(2e^{-2y})\,dy\,dx = \int_0^5 (1/5)(-e^{-2y})|_{y=x}^\infty\,dx = \int_0^5 (1/5)(e^{-2x})\,dx = (-1/10)(e^{-2x})|_{x=0}^5 = (1/10)(1 - e^{-10})$.

3. For $0 \le a \le 20$, we have $P(V \ge a) = P(X \ge a \;\&\; Y \ge a \;\&\; Z \ge a) = P(X \ge a)P(Y \ge a)P(Z \ge a) = (1 - a/20)^3$, so the CDF of V is $F_V(v) = P(V \le v) = 1 - P(V \ge v) = 1 - (1 - v/20)^3$, and it follows that the density of V is $f_V(v) = (3/20)(1 - v/20)^2$. So the expected value of V is $\mathbb{E}(V) = \int_0^{20} (v)(3/20)(1 - v/20)^2\,dv = \int_0^{20} (3/20)(v - v^2/10 + v^3/400)\,dv = (3/20)(v^2/2 - v^3/30 + v^4/1600)|_{v=0}^{20} = 5$.

4. We compute $\mathbb{E}(Y) = \int_0^5 \int_{y-5}^{5-y} (y)(1/25)\,dx\,dy = \int_0^5 (y)(1/25)(x)|_{x=y-5}^{5-y}\,dy = \int_0^5 (y)(1/25)((5-y) - (y-5))\,dy = \int_0^5 (y)(1/25)(10 - 2y)\,dy = \int_0^5 (1/25)(10y - 2y^2)\,dy = (1/25)(5y^2 - 2y^3/3)|_{y=0}^5 = 5/3$.

Alternatively, the density of Y is $f_Y(y) = \int_{y-5}^{5-y} (1/25)\,dx = (1/25)(x)|_{x=y-5}^{5-y} = (1/25)((5 - y) - (y - 5)) = (1/25)(10 - 2y)$ for $0 \le y \le 5$, and $f_Y(y) = 0$ otherwise. Then we get $\mathbb{E}(Y) = \int_0^5 (y)(1/25)(10 - 2y)\,dy$, and we proceed as we did in the paragraph above.