

STAT/MA 41600
In-Class Problem Set #33: October 31, 2018

1. Suppose that the time (in seconds) after midnight (on Halloween) until the Great Pumpkin appears is an exponential random variable X_1 , with $\mathbb{E}(X_1) = 15$. After the Great Pumpkin appears, suppose that the additional time it takes for children to see him is an exponential random variable X_2 , with $\mathbb{E}(X_2) = 15$. Also suppose that X_1 and X_2 are independent random variables. (I.e., the time it takes to see the Great Pumpkin after he arrives is *independent* of the time that it took him to arrive in the first place.)

Find $P(X_1 + X_2 > 30)$.

2. Suppose Dr. Ward sits on his front porch, with 700 pencils, ready to provide gifts to the first 700 children who come to Dr. Ward's home on Halloween night, for Trick-or-Treating.

Suppose that the time to wait for the next Trick-or-Treating visitor is always an exponential random variable, each with an average time (between visitors) of 5 seconds, and that these waiting times are independent.

2a. Consider the time until Dr. Ward is out of pencils. What kind of random variable is this? What is/are the parameter(s)?

2b. Find the expected value of the time until Dr. Ward is out of pencils.

2c. Find the variance of the time until Dr. Ward is out of pencils.

3. A couple is sitting on their front porch on Halloween night. Suppose the time X (in minutes) until the boyfriend scares the girlfriend is an exponential random variable, with a mean of 10 minutes. After this happens, suppose that the additional time Y (in minutes) until the girlfriend scares the boyfriend back (in retaliation) is an exponential random variable, with a mean of 3 minutes. Suppose that X and Y are independent random variables.

3a. Is $X + Y$ a Gamma random variable? Why or why not?

3b. Find $P(X + Y < 9)$.

4. On Halloween you are visiting a haunted corn maze. Scary creatures are running around everywhere.

Suppose that the time X (in seconds) until the first ghost materializes (out of thin air) is an exponential random variable with $\mathbb{E}(X) = 75$.

Suppose that the time Y (in seconds) until the first werewolf snarls (from the nearby eerie forest) is an exponential random variable with $\mathbb{E}(Y) = 35$.

Suppose that the time Z (in seconds) until the first bat swoops down (out of the darkness) is an exponential random variable with $\mathbb{E}(Z) = 50$.

Also suppose that X , Y , and Z are independent random variables.

You are terrified of ghosts, werewolves, and bats. Let V denote the time until the first of these scary things occurs.

4a. What kind of random variable is V ? What is/are the parameter(s)?

4b. Find the median of V (in seconds).