1a. The expected value is \( \alpha / (\alpha + \beta) = 6 / (6 + 2) = 6 / 8 = 3 / 4 \).

1b. The probability density function is \( f_X(x) = \frac{\Gamma(\alpha+2)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} \) for \( 0 \leq x \leq 1 \), and \( f_X(x) = 0 \) otherwise.

1c. We note \( f_X(x) \geq 0 \) for all \( x \), and \( f_0^1 42x^5(1-x) \, dx = f_0^1 42(x^5 - x^6) \, dx = 42(x^6/6 - x^7/7)|^1_x = 42(1/6 - 1/7) = 42(7/42 - 6/42) = 1 \).

2. We have \( P(X \leq 1/4) = f_0^{1/4} 42x^5(1-x) \, dx = f_0^{1/4} 42(x^5 - x^6) \, dx = 42(x^6/6 - x^7/7)|^{1/4}_x = 42((1/4)^6/6 - (1/4)^7/7) = 11/8192 \approx 0.001343 \).

3. We have \( P(U > X) = \int_0^1 \Gamma(\alpha+3/2)/\Gamma(\alpha+2) x^{\alpha-1} (1-x)^{\beta-1} \, dx \, du = \int_0^1 \int_0^u 12(x - 2x^2 + x^3) \, dx \, du = \int_0^1 12(x^2/2 - 2x^3/3 + x^4/4)|^u_x \, du = \int_0^1 12(u^2/2 - 2u^3/3 + u^4/4) \, du = 12(u^3/6 - u^4/6 + u^5/20)|^1_x = 3/5 \).

4. We compute \( P(X \geq Y) = \sum_{y=1}^\infty \sum_{x=y}^\infty (5/6)^{x-1}(1/2)^{y-1} = \sum_{y=1}^\infty (5/6)^{y-1}(1/2)(1/2)^{y-1} = \sum_{y=1}^\infty (5/6)^{y-1}(1/2)^{y-1} = (1/2)/(1 - 5/12) = 6/7 \).