STAT/MA 41600
In-Class Problem Set #35: November 5, 2018
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Problem Set 35 Answers

1. Let X denote the weight of the randomly chosen piece of chocolate. Let Z denote a standard Normal random variable.

We get $P(X > 1) = P\left(\frac{X-0.8}{.012} > \frac{1-0.8}{.012}\right) = P(Z > 1.67) = 1 - P(Z \leq 1.67) = 1 - 0.9525 = 0.0475$.

2. We let $p = 0.0475$. The number of pieces that exceed 1 ounce is a Binomial random variable with $n = 6$ and $p = 0.0475$, so the desired probability is $\binom{6}{3}p^3(1-p)^3 = 20(0.0475)^3(0.9525)^3 = 0.001852$.

3a. We compute $P(|X - 3| > .1) = P(X > 3.1 \text{ or } X < 2.9) = P(X > 3.1) + P(X < 2.9) = P\left(\frac{X-3}{.4} > \frac{3.1-3}{.4}\right) + P\left(\frac{X-3}{.4} < \frac{2.9-3}{.4}\right) = P(Z > 0.25) + P(Z < -0.25) = 2P(Z > 0.25) = 2(1 - P(Z \leq 0.25)) = 2(1 - .5987) = 0.8026$.

3b. We compute $P(|X - 2.8| > .1) = P(X > 2.9 \text{ or } X < 2.7) = P(X > 2.9) + P(X < 2.7) = P\left(\frac{X-3}{.4} > \frac{2.9-3}{.4}\right) + P\left(\frac{X-3}{.4} < \frac{2.7-3}{.4}\right) = P(Z > -0.25) + P(Z < -0.75) = P(Z < 0.25) + P(Z > 0.75) = P(Z < 0.25) + 1 - P(Z < 0.75) = 0.5987 + 1 - 0.7734 = 0.8253$.

4a. Let Y be the score of a randomly selected student. Then $P(Y > 80) = P\left(\frac{Y-72.5}{6.9} > \frac{80-72.5}{6.9}\right) = P(Z > 1.09) = 1 - P(Z \leq 1.09) = 1 - 0.8621 = 0.1379$.

4b. The random variable X is a geometric random variable with $p = 0.1379$, so $\mathbb{E}(X) = 1/p = 7.25$ and $\mathrm{Var}(X) = q/p^2 = 45.33$.