1. Suppose that $X$ and $Y$ are random variables with joint probability mass function

$$p_{X,Y}(x, y) = \frac{11}{16}(1/4)^{x-1}(1/3)^{y-1}$$

for integers $1 \leq y \leq x$.

Find $E(X \mid Y = 1)$, i.e., find the conditional expectation of $X$, given that $Y = 1$.

2. Suppose that $X$ and $Y$ have a constant joint probability density function on the triangle with vertices at $(0, 0)$, $(0, 6)$, $(10, 0)$, so that $f_{X,Y}(x, y) = 1/30$ for values $(x, y)$ in this triangle, and $f_{X,Y}(x, y) = 0$ otherwise.

Find $E(X \mid Y = 1)$, i.e., find the conditional expectation of $X$, given that $Y = 1$.

3. As in question 1 on Problem Set 26, suppose that the time (in seconds) until the next message arrives in Group Me is a continuous random variable $X$, and the time until the reply is denoted by $Y$. For this reason, we always have $Y > X$.

Suppose that the joint probability density function of $X$ and $Y$ is

$$f_{X,Y}(x, y) = \frac{1}{750}e^{-\left(x/150+y/30\right)}$$

for $y > x > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.

Given that $X = 20$, find the conditional expected value of $Y$.

4. Consider a 6-sided red die and a 6-sided blue die. Roll both of the dice. Given that the sum is exactly 9, find the conditional expected value of the value of the blue die.