STAT/MA 41600
In-Class Problem Set #42: November 30, 2018

1. Consider five independent exponential random variables (say, $X_1, \ldots, X_5$), each with expected value 2.

What is the probability that the minimum (i.e., $X_{(1)}$) is less than 1.5?

2. The bus arrives every 30 minutes at a certain stop, but students sometimes randomly run there, without even checking to see what time it is! This is the method by which Pat chooses to go to the bus stop every day. For this reason, assume that the time that Pat waits for the bus is always a continuous uniform random variable, distributed on the interval $[0, 30]$. Suppose that Pat has this behavior five days in a row, and we assume that the wait times are independent from day to day, so that X_1, X_2, X_3, X_4 X_5 are independent continuous random variables, each of which are uniformly distributed on $[0, 30]$.
2a. Find the probability that Pat never waits more than 20 minutes, during the five day week. I.e., find $P(X_{(5)} \leq 20)$.
2b. Find the probability that Pat has at least one day on which he waits 25 or more minutes. I.e., find $P(X_{(5)} \geq 25)$.
2c. What is the probability density function of $X_{(3)}$, namely, the third-shortest time that Pat has to wait?
2d. By symmetry, the expected value of $X_{(3)}$ should be exactly 15 (the midpoint of $[0, 30]$). Setup and evaluate an integral, using your answer from **2c**, to verify this.

3. Suppose that X_1, X_2, X_3 are independent random variables that each have probability density function $f_X(x) = x^2/9$ for $0 < x < 3$, and $f_X(x) = 0$ otherwise. What is the expected value of $X_{(3)}$, i.e., the expected value of the maximum of the three random variables?

4. (Review question) Draw 5 cards from a deck, without replacement. What is the variance of the number of diamonds that are chosen?