STAT/MA 41600
In-Class Problem Set #43: December 3, 2018
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Problem Set 43 Answers

1. We have $M_X(t) = \mathbb{E}(e^{tX}) = \sum_{x=1}^{\infty}(e^{tx})(3/5)(2/5)^{x-1} = (3/5)(e^t)\sum_{x=1}^{\infty}(e^{t(x-1)})(3/5)(2/5)^{x-1} = (3/5)(e^t)\sum_{x=1}^{\infty}(2e^t/5)^{x-1} = (3/5)(e^t)/(1-2e^t/5) = (3/5)/(e^{-t}-2/5)$.

2. We compute $M_X(t) = \mathbb{E}(e^{tX}) = \int_0^{\infty}(e^{tx})(7e^{-7x})\,dx = \int_0^{\infty}(7e^{(t-7)x})\,dx = 7e^{(t-7)x}/(t-7)|_{x=0}^{\infty} = 7/(7-t)$.

3a. We get $\mathbb{E}(X) = M_X'(0) = (3/5)(-1)(e^{-t}-2/5)^{-2}(-e^{-t})|_{t=0} = 5/3$.

3b. We get $\mathbb{E}(X) = M_X'(0) = (7)(-1)(7-t)^{-2}(-1)|_{t=0} = 1/7$.

4. Let $X_j = 1$ if the jth bear is happy, and $X_j = 0$ otherwise. Then $\mathbb{E}(X_1+\cdots+X_{18}) = \mathbb{E}(X_1)+\cdots+\mathbb{E}(X_{18}) = 18\mathbb{E}(X_1) = (18)(2/17)(1/16) = 9/68 = 0.1324$.