

STAT/MA 41600
In-Class Problem Set #16: September 26, 2014

- 1.** Matilda rolls a die until the first occurrence of “1,” and then she stops. Let X denote the number of rolls until (and including) that first occurrence of “1”; let Y denote the number of rolls that occur strictly before (but not including) that “1.” So we always have $Y = X - 1$.
 - 1a.** Find $\mathbb{E}(X)$.
 - 1b.** Find $\text{Var}(X)$.
 - 1c.** Find $\mathbb{E}(Y)$.
 - 1d.** Find $\text{Var}(Y)$.

- 2.** (Same problem setup as **(1)**.) She pays \$10 at the start, to play the game. Suppose Matilda wins \$1.25 for each die value unequal to “1”. (She doesn’t win anything for the “1” occurring at the end.) What is Matilda’s expected net gain or net loss from such a game?

- 3.** Let X be a geometric random variable with $\mathbb{E}(X) = 1/p$. Let a and b be fixed positive integers with $a < b$. Find $P(a \leq X \leq b)$.

- 4.** Let X be a geometric random variable with $\mathbb{E}(X) = 1/p$. Let a and b be fixed positive integers with $a < b$.
 - 4a.** Find the probability that $X > b$, given $X > a$, i.e., find $P(X > b \mid X > a)$.
 - 4b.** Find the probability that $X \leq b$, given $X > a$, i.e., find $P(X \leq b \mid X > a)$.
 - 4c.** Find the probability that $X = b$, given $X > a$, i.e., find $P(X = b \mid X > a)$.

- 5.** Suppose, on each round of a game, Alice rolls a 6-sided die, and Bob rolls a 4-sided die. They keep going until the first round on which they both (simultaneously) get values of 1, and then they stop. Let X denote the number of rolls until (and including) that first round on which they both (simultaneously) get 1.
 - 5a.** Find $\mathbb{E}(X)$.
 - 5b.** Find $\text{Var}(X)$.

- 6.** Suppose Alice rolls a (6-sided) dice until she gets her first occurrence of “1” and then she stops. Let X denote the number of rolls until (and including) that first occurrence of “1.” Suppose Bob flips a fair coin until he gets his first occurrence of “heads” and then he stops. Let Y denote the number of flips until (and including) that first occurrence of “heads.” Find $P(X \geq Y)$.