STAT/MA 41600
In-Class Problem Set #17: September 29, 2014

1. Matilda rolls a die until the eighth occurrence of "1," and then she stops. Let X denote the number of rolls until (and including) that eighth occurrence of "1"; let Y denote the number of rolls *not equal to "1"* that occur strictly before (but not including) that eighth "1." So we always have $Y = X - 8$.
1a. Find $\mathbb{E}(X)$.
1b. Find $\text{Var}\,(X)$.
1c. Find $\mathbb{E}(Y)$.
1d. Find $\text{Var}\,(Y)$.

2. (Same problem setup as **(1)**.) She pays \$54 at the start, to play the game. Suppose Matilda wins \$1.25 for each die value unequal to "1". (She doesn't win anything for any of the "1"s, throughout the game.) What is Matilda's expected net gain or net loss from such a game?

3. Let X be a negative binomial random variable with parameters r and p. Give a (relatively simple) expression for $P(X \leq r + 2)$.

4. Let X_1, X_2, X_3, X_4, X_5 be independent Geometric random variables, with $\mathbb{E}(X_j) = 3$ for each j. Define $X = X_1 + X_2 + X_3 + X_4 + X_5$.
4a. Find $P(X > 7 \mid X > 5)$.
4b. Find $P(X \leq 7 \mid X > 5)$.
4c. Find $P(X = 7 \mid X > 5)$.

5. At a certain college, 40% of the students live in a residence hall (on-campus), and the other 60% of the students live off-campus. Suppose that Audrey independently selects and interviews people, and she stops after she has found 6 students who live in a residence hall. Let X denote the number of interviews she conducts altogether.
5a. Find $\mathbb{E}(X)$.
5b. Find $\text{Var}\,(X)$.
5c. Find $P(X \geq 9)$.

6a. Suppose Alice rolls a 6-sided dice until she gets her eighth occurrence of "1" and then she stops. Let X denote the number of rolls until (and including) that eighth occurrence of "1." Find $\mathbb{E}(X)$ and $\text{Var}\,(X)$.
6b. Suppose Bob rolls a 6-sided dice until he gets his first occurrence of "1" and then he stops. Let Y denote the number of rolls until (and including) that first occurrence of "1." Let $Z = 8Y$. Find $\mathbb{E}(Z)$ and $\text{Var}\,(Z)$.
6c. Suppose Christine rolls an 8-sided dice until she gets her sixth occurrence of "1" and then she stops. Let U denote the number of rolls until (and including) that sixth occurrence of "1." Find $\mathbb{E}(U)$ and $\text{Var}\,(U)$.
6d. Suppose Daphne rolls an 8-sided dice until she gets her first occurrence of "1" and then she stops. Let V denote the number of rolls until (and including) that first occurrence of "1." Let $W = 6V$. Find $\mathbb{E}(W)$ and $\text{Var}\,(W)$.