STAT/MA 41600
In-Class Problem Set #25: October 17, 2014
Solutions by Mark Daniel Ward

1a. We have $P(X \geq 3) = \int_0^2 \int_3^{y+3} 1/8 \, dx \, dy = 1/4$.

1b. We have $P(Y \geq 1) = \int_1^2 \int_0^{y+3} 1/8 \, dx \, dy = 9/16$.

1c. We have $P(\max(X, Y) \leq 1) = \int_0^1 \int_0^1 1/8 \, dx \, dy = 1/8$.

2a. We have $P(X \leq 1) = \int_0^1 \int_0^{3-x} 2/9 \, dy \, dx = 5/9$.

2b. We have $P(Y \leq 2X) = \int_0^1 \int_0^{2x} 2/9 \, dy \, dx + \int_1^3 \int_0^{3-x} 2/9 \, dy \, dx = 2/9 + 4/9 = 2/3$.

3a. We have $P(Y \geq X) = \int_0^\infty \int_x^\infty 6e^{-3x-2y} \, dy \, dx = 3/5$.

3b. We have $P(X \geq 2) = \int_2^\infty \int_0^\infty 6e^{-3x-2y} \, dy \, dx = e^{-6}$.

3c. We have $P(Y \geq 5) = \int_0^\infty \int_5^\infty 6e^{-3x-2y} \, dy \, dx = e^{-10}$.

4a. We have $P(Y \geq \frac{1}{2}X + 1) = \int_{-2}^2 \int_{\frac{1}{2}x+1}^2 1/16 \, dy \, dx = 1/4$.

4b. We have $P(|X - 1| \leq 1/2) = P(-1/2 \leq X - 1 \leq 1/2) = P(1/2 \leq X \leq 3/2) = \int_{1/2}^{3/2} \int_{-2}^2 1/16 \, dy \, dx = 1/4$.

5a. The density of X is $f_X(x) = \frac{1}{8}(4 - x)$ for $0 < x < 4$, and $f_X(x) = 0$ otherwise.

5b. The density of Y is $f_Y(y) = \frac{2}{9}(3 - y)$ for $0 < y < 3$, and $f_Y(y) = 0$ otherwise.

6a. We have $P(Y \geq X) = \int_0^2 \int_0^y 1/8 \, dx \, dy = 1/4$.

6b. We have $P(Y \geq X) = \int_0^3 \int_x^3 \frac{1}{36}(4 - x)(3 - y) \, dy \, dx = 13/32$.