STAT/MA 41600
In-Class Problem Set #33: November 5, 2014
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1a. Since X is a Gamma random variable with $r = 2$ and $\lambda = 3$ then $\sigma_X = \sqrt{2/3^2} = \sqrt{2/9}$.
1b. The density of X is $f_X(x) = \frac{3^2}{(2-1)!}x^{2-1}e^{-3x} = 9xe^{-3x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.
1c. We have $P(X \leq 1) = F_X(1) = 1 - e^{-3(1)}\sum_{j=0}^{2-1}\frac{(3(1))^j}{j!} = 1 - e^{-3}(1+3) = 1 - 4e^{-3}$.

2a. We have $P(V = W) = 0$, so by symmetry, $P(V > W) = P(V < W) = 1/2$.
2b. We have $P(X \geq 1/2) = 1 - F_X(1/2) = e^{-3(1/2)}\sum_{j=0}^{2-1}\frac{(3(1/2))^j}{j!} = e^{-3/2}(1+\frac{3}{2}) = (5/2)e^{-3/2}$.

3a. Since V is a Gamma random variable with $r = 2$ and $\lambda = 5$, then the density of V is $f_V(v) = \frac{5^2}{(2-1)!}v^{2-1}e^{-5v} = 25ve^{-5v}$ for $v > 0$, and $f_V(v) = 0$ otherwise.
3b. We have $F_V(v) = 1 - e^{-5v}\sum_{j=0}^{2-1}\frac{(5v)^j}{j!} = 1 - e^{-5v}(1+5v)$ for $v > 0$; otherwise, $F_V(v) = 0$.

4a. We have $\mathrm{Var}(V) = 2/5^2 = 2/25$.
4b. We have $P(V \leq 1/5) = F_V(1/5) = 1 - e^{-5(1/5)}(1 + 5(1/5)) = 1 - 2e^{-1}$.

5. We have $P(Y > X) = \sum_{x=0}^{\infty}\sum_{y=x+1}^{\infty}p(1-p)^{y-1}e^{-\lambda}\lambda^x/x! = \sum_{x=0}^{\infty}pe^{-\lambda}\lambda^x/x!\sum_{y=x+1}^{\infty}(1-p)^{y-1} = \sum_{x=0}^{\infty}e^{-\lambda}\lambda^x(1-p)^x/x! = e^{-\lambda}e^{\lambda(1-p)} = e^{-\lambda p}$.

6a. We have $P(Y \geq 1) = P(X \geq 1) = e^{-3}$.
6b. We have $P(Y \geq 5) = P(X \geq 5) = e^{-15}$.
6c. We have $P(Y \geq 10) = P(X \geq 10) = e^{-30}$.
6d. We have $P(Y \geq x) = P(X \geq x) = e^{-3x}$.
6e. We have $P(Y = x) = P(Y \geq x) - P(Y \geq x+1) = e^{-3x} - e^{-3(x+1)} = e^{-3x}(1 - e^{-3})$ for $x \geq 0$. Thus, Y is a Geometric "number of losses" random variable with $p = 1 - e^{-3}$ and $q = e^{-3}$.