STAT/MA 41600
In-Class Problem Set #36: November 12, 2014
Solutions by Mark Daniel Ward

1a. We have $P(Y < 42.5) = P(\frac{Y-10(4)}{\sqrt{10(0.75)^2}} < \frac{42.5-10(4)}{\sqrt{10(0.75)^2}}) = P(Z < 1.05) = 0.8531$.
1b. We have $P(39.5 < Y < 40.5) = P(Y < 40.5) - P(Y < 39.5) = P(\frac{Y-10(4)}{\sqrt{10(0.75)^2}} < \frac{40.5-10(4)}{\sqrt{10(0.75)^2}}) - P(\frac{Y-10(4)}{\sqrt{10(0.75)^2}} < \frac{39.5-10(4)}{\sqrt{10(0.75)^2}}) = P(Z < 0.21) - P(Z < -0.21) = 0.5832 - (1 - 0.5832) = 0.1664$.

2a. We have $P(6350 < X < 6450) = P(\frac{6350-100(64)}{\sqrt{100(4.8)^2}} < \frac{X-100(64)}{\sqrt{100(4.8)^2}} < \frac{6450-100(64)}{\sqrt{100(4.8)^2}}) = P(-1.04 < Z < 1.04) = P(Z < 1.04) - (1 - P(Z < 1.04)) = 0.8508 - (1 - 0.8508) = 0.7016$.
2b. We have $P(6390 < X < 6410) = P(\frac{6390-100(64)}{\sqrt{100(4.8)^2}} < \frac{X-100(64)}{\sqrt{100(4.8)^2}} < \frac{6410-100(64)}{\sqrt{100(4.8)^2}}) = P(-0.21 < Z < 0.21) = P(Z < 0.21) - (1 - P(Z < 0.21)) = 0.5832 - (1 - 0.5832) = 0.1664$.

3a. We have $P(2750 < X < 2850) = P(\frac{2750-40(70)}{\sqrt{40(81)}} < \frac{X-40(70)}{\sqrt{40(81)}} < \frac{2850-40(70)}{\sqrt{40(81)}}) = P(-0.88 < Z < 0.88) = P(Z < 0.88) - (1 - P(Z < 0.88)) = 0.8106 - (1 - 0.8106) = 0.6212$.
3b. We have $P(|Y - \mathbb{E}(Y)| < \sigma_Y) = P(\mathbb{E}(Y) - \sigma_Y < Y < \mathbb{E}(Y) + \sigma_Y) = P(\frac{\mathbb{E}(Y)-\sigma_Y-\mathbb{E}(Y)}{\sigma_Y} < Y < \frac{\mathbb{E}(Y)+\sigma_Y-\mathbb{E}(Y)}{\sigma_Y}) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1) = P(Z < 1) - (1 - P(Z < 1)) = 0.8413 - (1 - 0.8413) = 0.6826$.

4a. We have $P(Y < X < Y + 3) = P(0 < X - Y < 3) = P(\frac{0-(75-70)}{\sqrt{50+60}} < \frac{X-Y-(75-70)}{\sqrt{50+60}} < \frac{3-(75-70)}{\sqrt{50+60}}) = P(-0.48 < Z < -0.19) = P(Z < -0.19) - P(Z < -0.48) = P(Z > 0.19) - P(Z > 0.48) = (1-P(Z < 0.19)) - (1-P(Z < 0.48)) = (1-0.5753) - (1-0.6844) = 0.1091$.
4b. Yes, it is possible. We have $P(X < Y) = P(X - Y < 0) = P(\frac{X-Y-(75-70)}{\sqrt{50+60}} < \frac{0-(75-70)}{\sqrt{50+60}} = P(Z < -0.48) = P(Z > 0.48) = 1 - P(Z < 0.48) = 1 - 0.6844 = 0.3156$.

5a. Let X denote the sum of the weights of the cars, and Y denote the weight of the puzzle, we have $P(X > Y) = P(X - Y > 0) = P(\frac{X-Y-(10(35)-400)}{\sqrt{10(12^2)+75^2}} > \frac{0-(10(35)-400)}{\sqrt{10(12^2)+75^2}}) = P(Z > 0.59) = 1 - P(Z < 0.59) = 1 - 0.7224 = 0.2776$.
5b. We have $P(X < 325) = P(\frac{X-10(35)}{\sqrt{10(12^2)}} < \frac{325-10(35)}{\sqrt{10(12^2)}}) = P(Z < -0.66) = P(Z > 0.66) = 1 - P(Z < 0.66) = 1 - 0.7454 = .2546$.

6a. We have $P(X_1 + \cdots + X_{200} < 700) = P(\frac{X_1+\cdots+X_{200}-200(3)}{\sqrt{200(25)}} < \frac{700-200(3)}{\sqrt{200(25)}}) = P(Z < 1.41) = 0.9207$.
6b. We have $P(|Y - 3| \leq 0.2) = P(2.8 \leq Y \leq 3.2) = P(2.8 \leq (X_1 + \cdots + X_{200})/200 \leq 3.2) = P(560 \leq X_1 + \cdots + X_{200} \leq 640) = P(\frac{560-200(3)}{\sqrt{200(25)}} \leq \frac{X_1+\cdots+X_{200}-200(3)}{\sqrt{200(25)}} \leq \frac{640-200(3)}{\sqrt{200(25)}} = P(-0.57 \leq Z \leq 0.57) = P(Z \leq 0.57) - P(Z \leq -0.57) = P(Z \leq 0.57) - (1-P(Z \leq 0.57)) = 0.7157 - (1 - 0.7157) = 0.4314$.