Roll a 6-sided die repeatedly until we get a specified value, say, "3."

Let X be the # of rolls needed. Find $E(X)$.

$$E(X) = (1)(\tfrac{1}{6}) + (2)(\tfrac{5}{6})(\tfrac{1}{6}) + (3)(\tfrac{5}{6})^2(\tfrac{1}{6}) + (4)(\tfrac{5}{6})^3(\tfrac{1}{6}) + (5)(\tfrac{5}{6})^4(\tfrac{1}{6}) + \cdots$$

$$= \sum_{j=1}^{\infty} (j)(\tfrac{5}{6})^{j-1}(\tfrac{1}{6})$$

Idea: $$j(\tfrac{5}{6})^{j-1} = j x^{j-1}\Big|_{x=5/6} = \frac{d}{dx} x^j \Big|_{x=5/6}$$

$$= \frac{1}{6} \sum_{j=1}^{\infty} \frac{d}{dx} x^j \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{d}{dx} \left(\sum_{j=1}^{\infty} x^j \right) \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{d}{dx} \frac{x}{1-x} \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{1}{(1-x)^2} \Big|_{x=5/6}$$

$$= \frac{1}{6} \frac{1}{(1/6)^2} = \boxed{6}$$