Example of geometric random variables. My daughter Audrey is left-handed. She says that 10% of people are left-handed. Suppose that we talk to people until we find the first person who is left-handed.

So we define X as the number of people required until (and including) the first left-handed person appears. We assume that the people's attributes are independent. For instance, if we meet R, R, R, R, R, R, L, then $X = 7$.

Let's find $E(X)$, assuming the $p = 1/10$ throughout.

To do this, we define $X_j = 1$ if j or more people are needed, to get the first left-handed person; otherwise $X_j = 0$. So X_j is an indicator. In our example, $X_1 = X_2 = \ldots = X_7 = 1$, and $X_8 = X_9 = X_{10} = \ldots = 0$. We saw this idea one other time in an earlier lesson.

Notice that $E(X_j) = P(X_j = 1)$ since X_j is an indicator. Also $X_j = 1$ if and only if the first $j - 1$ trials are all failures. So $E(X_j) = P(X_j = 1) = q^{j-1}$, or in this case, $p = 1/10$ so $q = 9/10$, so $E(X_j) = (9/10)^{j-1}$.

So

$$E(X) = E(X_1+X_2+X_3+\ldots) = E(X_1)+E(X_2)+\ldots = 1+9/10+(9/10)^2+\ldots = \frac{1}{1-9/10} = 10$$

So we expect to need to meet 10 people in order to find the first person who is left-handed.

The same idea works in general for Geometric(p) random variables. If X is Geometric(p) random variable, and we define $X_j = 1$ if $X \geq j$, and $X_j = 0$ otherwise, then

$$E(X) = E(X_1 + X_2 + \ldots) = E(X_1) + E(X_2) + \ldots = 1 + q + q^2 + q^3 + \ldots = \frac{1}{1-q} = 1/p.$$

We will need a different method to get the variance of Geometric random variables, but this is a nice way to think about the expected values of Geometric random variables.