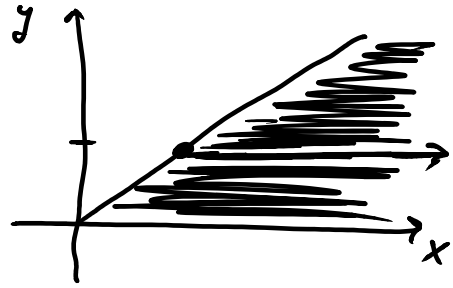


Example Suppose X and Y have joint density $f_{X,Y}(x,y) = \begin{cases} 6e^{-2x-3y} & x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$

So in particular, X and Y are both positive.

Find $P(X > Y) = \iint_{x>y} f_{X,Y}(x,y) dy dx$
↑
haven't yet determined
how to specify the boundaries
or $dx dy$



$$= \int_0^{\infty} \int_y^{\infty} 6e^{-2x-3y} dx dy$$

$$= \int_0^{\infty} \left. \frac{6e^{-2x-3y}}{-2} \right|_{x=y}^{\infty} dy$$

$$= \int_0^{\infty} 3e^{-2y-3y} dy$$

$$= \left. \frac{3e^{-5y}}{-5} \right|_{y=0}^{\infty}$$

$$= \boxed{\frac{3}{5}}$$

So $P(X > Y) = \frac{3}{5}$.

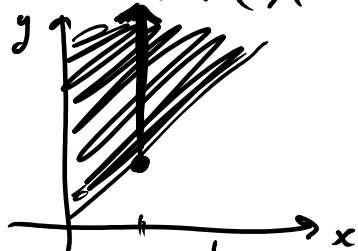
Q: $P(X=Y)$??? Must be 0!
 $P(X > Y) + P(X=Y) + P(X < Y) = 1$
 $\frac{3}{5} + 0 + \frac{2}{5} = 1$

Check directly:

$$P(X=Y) = \int_0^{\infty} \int_x^x 6e^{-2x-3y} dy dx = \int_0^{\infty} 0 dx = 0.$$

This is a common phenomena for continuous random variables.
 The probability of exact ties is often 0.

Now find $P(X < Y)$



$$P(X < Y) = \int_0^{\infty} \int_x^{\infty} 6e^{-2x-3y} dy dx$$

$$= \int_0^{\infty} \left. \frac{6e^{-2x-3y}}{-3} \right|_{y=x}^{\infty} dx$$

$$= \int_0^{\infty} 2e^{-2x-3x} dx$$

$$= \left. \frac{2e^{-5x}}{-5} \right|_{x=0}^{\infty}$$

$$= \boxed{\frac{2}{5}} \quad \text{So } P(X < Y) = \frac{2}{5}$$