

Use the joint density of X and Y to find the single variable density of X or of Y . (Sometimes called the marginal density of X or Y .)

In general if X and Y have joint density $f_{X,Y}(x,y)$

then X has density $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy$ i.e. think: integrating y out of the picture.

and Y has density $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$ i.e. integrate x out of the picture.

Example Say X, Y have joint density $f_{X,Y}(x,y) = \begin{cases} 6e^{-2x-3y} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$

Then X has density $f_X(x) = 0$ if $x < 0$

$$\begin{aligned} \text{or if } x > 0: f_X(x) &= \int_0^{\infty} 6e^{-2x-3y} dy = \frac{6e^{-2x-3y}}{-3} \Big|_{y=0}^{\infty} \\ &= 2e^{-2x} \text{ (for } x > 0) \end{aligned}$$

Also Y has density: $f_Y(y) = 0$ if $y < 0$

$$\begin{aligned} \text{or if } y > 0: f_Y(y) &= \int_0^{\infty} 6e^{-2x-3y} dx = \frac{6e^{-2x-3y}}{-2} \Big|_{x=0}^{\infty} = 3e^{-3y} \\ &\text{(for } y > 0) \end{aligned}$$

This method works in general if you have the joint density of X and Y but just want the single variable density of X or of Y .