

Another nice fact: We know that $E(aX) = aE(X)$ for constant a . Why?

$$E(aX) = \int_{-\infty}^{\infty} ax f_X(x) dx = a \int_{-\infty}^{\infty} x f_X(x) dx = aE(X).$$

Another nice idea:

$$E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n).$$

Why is that? We just apply the rule $E(X + Y) = E(X) + E(Y)$ over and over again, until all of the n terms are separated. In other words, the first time, you treat X_1 as X and $X_2 + \cdots + X_n = Y$, and we get $E(X_1 + X_2 + \cdots + X_n) = E(X_1) + E(X_2 + \cdots + X_n)$. So you pull the $E(X_1)$ term off, in other words. Then do it again for pulling off $E(X_2)$, etc., etc.

Another nice consequence of these two facts is:

$$E(a_1X_1 + \cdots + a_nX_n) = E(a_1X_1) + \cdots + E(a_nX_n) = a_1E(X_1) + \cdots + a_nE(X_n),$$

for any constants a_1, \dots, a_n .