Example: Suppose we left the office at 1:15 PM, and returned at 1:25 PM, and find 1 phone message on the telephone machine, when we return. Assume the time that call arrives has a uniform continuous distribution on the interval from the start of the time period to the end of the time period. Let X denote the time (in minutes) after 1 PM when the call arrived. So X is continuous uniform random variable on the interval $[15, 25]$.

The density of X is $f_X(x) = \frac{1}{25-15} = 1/10$ for $15 < x < 25$, and $f_X(x) = 0$ otherwise.

The CDF of X is $F_X(x) = 0$ for $x \leq 15$, and $F_X(x) = 1$ for $x \geq 25$, and $F_X(x) = \frac{x-15}{25-15}$ for $15 < x < 25$.

What about the expected value of X? We have $E(X) = \frac{15+25}{2} = 40/2 = 20$, i.e., the expected time the call arrived would be 1:20 PM.

What about the variance? The variance is $Var(X) = \frac{(25-15)^2}{12} = 100/12 = 25/3$.

For instance, what is the probability that the call arrived before 1:17 PM? One method is to integrate: $P(X \leq 17) = \int_{15}^{17} \frac{1}{25-15}\,dx = \frac{17-15}{25-15} = 2/10 = 1/5$. Another method is to recognize that we are just integrating a constant over some interval, so the integral is the length of the integration path, times the integrand, which is $1/(b-a)$, i.e., which is 1 divided by the length of the interval where X is defined. So we can simply take the length of $[15, 17]$ and divide by the length of the whole interval where X is defined, $[15, 25]$, and we get $P(X \leq 17) = 2/10 = 1/5$, as we determined before.

There is nothing special about this case. Whenever we have a continuous uniform random variable, say X, the probability that X is in some region (within the realm where X is defined), is the length of that region divided by the whole length where X is defined.

For another example, $P(21 \leq X \leq 24) = \text{length of } [21, 24]/\text{length of } [15, 25] = 3/10$.