

Example Let Y be a Gamma random variable with $r=1000$, $\lambda=8$.

Find $P(620 < Y < 630)$. Why is this a CLT problem??

Notice Y has the same distribution as $X_1 + X_2 + \dots + X_{1000}$ where the X_j 's are independent exponential random variables, each with mean $\mu = \frac{1}{8}$ and variance $\sigma^2 = \frac{1}{8^2}$.

$$\text{So } P(620 < Y < 630) = P(620 < X_1 + \dots + X_{1000} < 630)$$

$$\begin{aligned} \text{by CLT } \left\{ \begin{aligned} &= P\left(\frac{620 - (1000)(\frac{1}{8})}{\sqrt{1000}(\frac{1}{8^2})} < \frac{X_1 + \dots + X_{1000} - (1000)(\frac{1}{8})}{\sqrt{1000}(\frac{1}{8^2})} < \frac{630 - (1000)(\frac{1}{8})}{\sqrt{1000}(\frac{1}{8^2})}\right) \\ &\approx P(-.57 < Z < .57) \\ &= P(Z < .57) - P(Z < -.57) \\ &= 0.7157 - (1 - 0.7157) \\ &= 0.4314 \end{aligned} \right. \end{aligned}$$