Variance of the sum of some random variables:

$$\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{n}\Big(\underbrace{E(X_iX_j) - E(X_i)E(X_j)}_{\mathrm{Cov}(X_i, X_j)}\Big)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(X_i, X_j)$$

Also know $\mathrm{Var}(X) = \mathrm{Cov}(X, X)$

E.g.

$$\mathrm{Var}(X_1 + X_2 + X_3) = \sum_{i=1}^{3}\sum_{j=1}^{3} \mathrm{Cov}(X_i, X_j) = \mathrm{Cov}(X_1,X_1) + \mathrm{Cov}(X_1,X_2) + \mathrm{Cov}(X_1,X_3)$$
$$+ \mathrm{Cov}(X_2,X_1) + \mathrm{Cov}(X_2,X_2) + \mathrm{Cov}(X_2,X_3)$$
$$+ \mathrm{Cov}(X_3,X_1) + \mathrm{Cov}(X_3,X_2) + \mathrm{Cov}(X_3,X_3)$$
$$= \sum_{i=1}^{3}\mathrm{Var}(X_i) + \sum_{i=1}^{3}\sum_{\substack{j=1\\ j\neq i}}^{3} \mathrm{Cov}(X_i, X_j)$$
$$= \sum_{i=1}^{3}\mathrm{Var}(X_i) + 2\sum\sum_{1\le i<j\le 3} \mathrm{Cov}(X_i, X_j)$$

$$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$$
$$= E(YX) - E(Y)E(X)$$
$$= \mathrm{Cov}(Y,X)$$

More generally, at least 3 ways to write the variance of the sum of $n$ random variables $X_1, \dots, X_n$:

$$\mathrm{Var}(X_1 + \dots + X_n) = \sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{Cov}(X_i, X_j)$$

$$\mathrm{Var}(X_1 + \dots + X_n) = \sum_{i=1}^{n}\mathrm{Var}(X_i) + \sum_{i=1}^{n}\sum_{j\neq i} \mathrm{Cov}(X_i, X_j)$$

$$\mathrm{Var}(X_1 + \dots + X_n) = \sum_{i=1}^{n}\mathrm{Var}(X_i) + 2\sum_{i<j} \mathrm{Cov}(X_i, X_j)$$

3 equivalent ways of computing the variance of $X_1 + \dots + X_n$