STAT/MA 41600
Practice Problems: August 27, 2014
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**Problem Set 1 Answers**

**1. Choosing points at random.** (a.) The sample space is

$$S = \{(x, y) \mid 0 \le x \le 2,\ 0 \le y \le 2 - x\}.$$

(b.) The sample space is

$$S = \{(x, y) \mid 0 \le x \le 2,\ 0 \le y \le 4 - x\}.$$

**2. Gloves.** (a.) Any of the 5 gloves can come out first, followed by any of the 4 remaining gloves, followed by any of the 3 remaining gloves, followed by any of the 2 remaining gloves, followed by the last glove. So there are $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$ possible outcomes.

(b.) Each outcome in part (b) represents exactly $2 \times 2 = 4$ of the outcomes from part (a). For instance, this outcome from part (b):

(“blue”, “white”, “red”, “red”, “blue”).

corresponds to these four outcomes from part (a):

(“blue right”, “white right”, “red left”, “red right”, “blue left”),

(“blue left, “white right”, “red left”, “red right”, “blue right),

(“blue right”, “white right”, “red right”, “red left”, “blue left”),

(“blue left, “white right”, “red right”, “red left”, “blue right).

So there are only 1/4 as many outcomes in part (b), as compared to part (a). Thus there are $\frac{120}{4} = 30$ outcomes possible in part (b).

**3. Seating arrangements.**

*Method #1:* Alice sits somewhere. Someone, say a “mystery” person, sits on Alice’s right-hand side. Regardless of who this mystery person is, the mystery person will be sitting next to her/his spouse in one of three possible ways (and of course the other couple will be together in such a case too). So the couples are happy in 1 out of every 3 possible arrangements, i.e., in $24/3 = 8$ of the possible arrangements.

*Method #2:* Alice sits somewhere. For the couples to sit together, either Bob and Catherine (collectively, as a pair) sit on Alice’s left or right. In either case, Bob and Catherine have two ways of sitting among themselves (i.e., within their own pair). Within the two remaining seats, Doug and Edna have two ways of sitting among themselves (i.e., within their own pair). So there are $2 \times 2 \times 2 = 8$ possible ways that the couples can sit happily.

**4. Abstract art.**

An event is completely known once we decide whether or not each outcome belongs in the event. So, for each of the five outcomes, we just decide ("yes" or "no") whether that outcome goes in the event we are building. So there are $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$ possible events that we could build.

**5. Sum of three dice.** We have:

| event | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ | $A_{10}$ |
|---|---|---|---|---|---|---|---|---|
| number of outcomes | 1 | 3 | 6 | 10 | 15 | 21 | 25 | 27 |

| event | $A_{11}$ | $A_{12}$ | $A_{13}$ | $A_{14}$ | $A_{15}$ | $A_{16}$ | $A_{17}$ | $A_{18}$ |
|---|---|---|---|---|---|---|---|---|
| number of outcomes | 27 | 25 | 21 | 15 | 10 | 6 | 3 | 1 |

Perhaps the easiest way to solve this problem is to go through the possible choices of values for the first die, and then use the chart suggested, to see how many outcomes are possible for the second and third die.

For instance, to determine $A_{15}$, we could have:
If the first die is 1 or 2, the second and third dice cannot be large enough for a sum of 15.
If the first die is 3, there is 1 way the second and third dice add up to 12.
If the first die is 4, there are 2 ways the second and third dice add up to 11.
If the first die is 5, there are 3 ways the second and third dice add up to 10.
If the first die is 6, there are 4 ways the second and third dice add up to 9.

So $A_{15}$ has $1 + 2 + 3 + 4 = 10$ possible outcomes.

The problem is completely symmetric too, so once you have done $A_{11}$ through $A_{18}$, the other counts of outcomes are the same, in reverse order.