

STAT/MA 41600  
Practice Problems #2: November 14, 2014  
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1. Let  $X$  be the number of flights that are on time. Then  $X$  is Binomial with  $n = 2000$  and  $p = 0.70$ , so  $P(X > 1420) = P(X > 1420.5) = P\left(\frac{X - (2000)(0.70)}{\sqrt{(2000)(0.70)(0.30)}} > \frac{1420.5 - (2000)(0.70)}{\sqrt{(2000)(0.70)(0.30)}}\right) \approx P(Z > 1.00) = 1 - P(Z \leq 1.00) = 1 - .8413 = 0.1587$ .

2. Let  $X$  be the number of students who attend.

Then  $X$  is a Binomial random variable with  $n = 400$ ,  $p = 0.60$ , so  $P(230 \leq X \leq 250) = P(229.5 \leq X \leq 250.5) = P\left(\frac{229.5 - (400)(0.60)}{\sqrt{(400)(0.60)(0.40)}} \leq \frac{X - (400)(0.60)}{\sqrt{(400)(0.60)(0.40)}} \leq \frac{250.5 - (400)(0.60)}{\sqrt{(400)(0.60)(0.40)}}\right)$ . This is approximately  $P(-1.07 \leq Z \leq 1.07) = P(Z \leq 1.07) - P(Z < -1.07) = P(Z \leq 1.07) - P(Z > 1.07) = P(Z \leq 1.07) - (1 - P(Z \leq 1.07)) = 2P(Z \leq 1.07) - 1 = 2(.8577) - 1 = .7154$ .

3. Let  $X$  be the number of broken crayons.

Then  $X$  is a Binomial random variable,  $n = 12,000$ ,  $p = 0.05$ , so  $P(580 \leq X \leq 620) = P(579.5 \leq X \leq 620.5) = P\left(\frac{579.5 - (12,000)(0.05)}{\sqrt{(12,000)(0.05)(0.95)}} \leq \frac{X - (12,000)(0.05)}{\sqrt{(12,000)(0.05)(0.95)}} \leq \frac{620.5 - (12,000)(0.05)}{\sqrt{(12,000)(0.05)(0.95)}}\right)$ . This is roughly  $P(-0.86 \leq Z \leq 0.86) = P(Z \leq 0.86) - P(Z < -0.86) = P(Z \leq 0.86) - P(Z > 0.86) = P(Z \leq 0.86) - (1 - P(Z \leq 0.86)) = 2P(Z \leq 0.86) - 1 = 2(.8051) - 1 = .6102$ .

4. Let  $X$  be the number of passengers with the extra screening.

Then  $X$  is a Binomial random variable with  $n = (8)(180) = 1440$  and  $p = 0.06$ , so  $P(X \geq 80) = P(X \geq 79.5) = P\left(\frac{X - (1440)(0.06)}{\sqrt{(1440)(0.06)(0.94)}} \geq \frac{79.5 - (1440)(0.06)}{\sqrt{(1440)(0.06)(0.94)}}\right) \approx P(Z \geq -0.77) = P(Z \leq 0.77) = .7794$ .

5. Let  $X$  be the number of field goals Jeff makes successfully. Let  $Y$  be the number of field goals Steve makes successfully. So we want  $P(X > Y)$ , i.e.,  $P(X - Y > 0)$ . We see that

$$X - Y = X_1 + X_2 + \cdots + X_{120} - Y_1 - Y_2 - \cdots - Y_{164},$$

where  $X_j$  indicates whether Jeff's  $j$ th attempt was a success, and  $Y_j$  indicates whether Steve's  $j$ th attempt was a success. So  $X - Y$  is the sum of a large number of independent random variables, and thus  $X - Y$  is approximately normal.

We have  $\mathbb{E}(X - Y) = (120)(.80) - (164)(.60) = -2.40$ , and  $\text{Var } X - Y = \text{Var } X + \text{Var } Y = (120)(.80)(.20) + (164)(.60)(.40) = 58.56$ . Thus  $P(X > Y) = P(X - Y > 0) = P(X - Y > 0.5) = P\left(\frac{X - Y - (-2.40)}{\sqrt{58.56}} > \frac{0.5 - (-2.40)}{\sqrt{58.56}}\right) \approx P(Z > 0.38) = 1 - P(Z \leq 0.38) = 1 - .6480 = .3520$ .