

STAT/MA 41600  
In-Class Problem Set #28: October 23, 2015

1. Suppose  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = 70e^{-3x-7y}$$

for  $0 < x < y$ ; and  $f_{X,Y}(x, y) = 0$  otherwise. Find  $\mathbb{E}(X)$ . (You may either use the joint density given here, or the density  $f_X(x)$  that was found in **1a** of the previous problem set.)

2. For the setup in question **1**, find  $\mathbb{E}(Y)$ . (In this example, there are tradeoffs to the order of integration that you choose to use, i.e., to whether you integrate with respect to  $x$  or  $y$  first. You might find it instructive to try it both ways and compare the difficulties; this would also enable you to double-check your answer.)

3. Consider a pair of random variables  $X, Y$  with constant joint density on the triangle with vertices at  $(0, 0)$ ,  $(2, 0)$ , and  $(0, 8)$ . Find  $\mathbb{E}(X)$ .

**4a.** Suppose that  $Y$  is an exponential random variable with probability density function  $f_Y(y) = 5e^{-5y}$  for  $y > 0$ , and  $f_Y(y) = 0$  otherwise. Compute  $\mathbb{E}(Y)$ .

**4b.** Generalize the result in **4a**. In other words, suppose that  $\lambda > 0$  is a fixed constant, and suppose that  $Y$  is an exponential random variable with probability density function  $f_Y(y) = \lambda e^{-\lambda y}$  for  $y > 0$ , and  $f_Y(y) = 0$  otherwise. Compute  $\mathbb{E}(Y)$ .