STAT/MA 41600
In-Class Problem Set #17: September 26, 2016
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Problem Set 17 Answers

1a. Let X denote the number of people called. Then $P(X \geq 12) = 1 - P(X \leq 11) = 1 - P(X = 10) - P(X = 11) = 1 - \binom{9}{9}(8/10)^{10} - \binom{10}{9}(8/10)^{10}(2/10) = 1 - (8/10)^{10} - 10(8/10)^{10}(2/10) = 6619897/9765625 = 0.6779$.
1b. We have $P(X \geq 14 \mid X \geq 12) = \frac{P(X\geq 14 \ \& \ X\geq 12)}{P(X\geq 12)} = \frac{P(X\geq 14)}{P(X\geq 12)}$, so $P(X \geq 14 \mid X \geq 12) = \frac{1-P(X=10)-P(X=11)-P(X=12)-P(X=13)}{1-P(X=10)-P(X=11)} = \frac{1-(8/10)^{10}-10(8/10)^{10}(2/10)-\binom{11}{2}(8/10)^{10}(2/10)^2-\binom{12}{3}(8/10)^{10}(2/10)^3}{1-(8/10)^{10}-10(8/10)^{10}(2/10)} = \frac{61688401/244140625}{6619897/9765625} = 61688401/165497425 = 0.3727$.
1c. We have $\mathbb{E}(X) = (10)(1/(8/10)) = 25/2$.
1d. We have $\mathrm{Var}\,(X) = (10)((2/10)/(8/10)^2) = 25/8$.

2a. We have $\mathbb{E}(X_1 + X_2 + X_3 + X_4) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \mathbb{E}(X_4) = 5/2 + 5/2 + 5/2 + 5/2 = 10$, and $\mathbb{E}(Y) = 4/p = (4)/(2/5) = 10$, and $\mathbb{E}(Z) = \mathbb{E}(4X_1) = 4\mathbb{E}(X_1) = (4)(5/2) = 10$.
2b. Since the X_i's are independent, we have $\mathrm{Var}\,(X_1 + X_2 + X_3 + X_4) = \mathrm{Var}\,(X_1) + \mathrm{Var}\,(X_2) + \mathrm{Var}\,(X_3) + \mathrm{Var}\,(X_4) = (3/5)/(2/5)^2 + (3/5)/(2/5)^2 + (3/5)/(2/5)^2 + (3/5)/(2/5)^2 = 15$, and $\mathrm{Var}\,(Y) = 4q/p^2 = (4)(3/5)/(2/5)^2 = 15$, and $\mathrm{Var}\,(Z) = \mathrm{Var}\,(4X_1) = 4^2\,\mathrm{Var}\,(X_1) = (4^2)(3/5)/(2/5)^2 = 60$.

3a. The random variable $U + V$ is not a Negative Binomial random variable because $p = 1/6$ for U and $p = 1/4$ for V.
3b. We note that X is a Negative Binomial random variable with $r = 2$ and $p = 1/6$ so the probability mass function is $p_X(x) = P(X = x) = \binom{x-1}{2-1}(1/6)^2(5/6)^{x-2}$.

4a. We have $P(X \text{ is even}) = \sum_{j=1}^{\infty}(1/2)^{2j} = \sum_{j=1}^{\infty}(1/4)^j = (1/4)/(1 - 1/4) = 1/3$.
4b. We have $P(X \text{ is a multiple of } 3) = \sum_{j=1}^{\infty}(1/2)^{3j} = \sum_{j=1}^{\infty}(1/8)^j = (1/8)/(1-1/8) = 1/7$.
4c. We have $P(X \text{ is a multiple of } 4) = \sum_{j=1}^{\infty}(1/2)^{4j} = \sum_{j=1}^{\infty}(1/16)^j = (1/16)/(1 - 1/16) = 1/15$.
4d. In general, we compute $P(X \text{ is a multiple of n}) = \sum_{j=1}^{\infty}(1/2)^{4n} = \sum_{j=1}^{\infty}((1/2)^n)^j = (1/2)^n/(1 - (1/2)^n) = 1/(2^n - 1)$.