STAT/MA 41600
In-Class Problem Set #33: November 2, 2016

1. Revisit the following three questions from problem set 32 part 2 (from Monday):
1a. In question 1a, we considered the time X until the full moon appears, and the time Y until the next wolf howls. Is $X+Y$ a Gamma random variable? If so, what are the parameters of $X+Y$? If not, then why not?
1b. In question 3, we considered the (respective) times V and W until we saw the next vampire and werewolf. Is $V+W$ a Gamma random variable? If so, what are the parameters of $V+W$? If not, then why not?
1c. In question 4c, we considered the lifetimes of n pumpkins, which we will call $X_1, \ldots, X_n$. Is $X_1+\cdots+X_n$ a Gamma random variable? If so, what are the parameters of $X_1+\cdots+X_n$? If not, then why not?

2. Suppose that the time X (in minutes) until the next set of commercials starts is an exponential random variable with expected value 2. Suppose that the length Y (in minutes) of the commercials themselves is an exponential random variable with expected value 1.5. Assume X and Y are independent.
2a. Is $X+Y$ is a Gamma random variable? If so, what are the parameters of $X+Y$? If not, then why not?
2b. Find the standard deviation of $X+Y$.

3. Suppose that X and Y have joint probability density function $f_{X,Y}(x,y)=16e^{-4x-4y}$ for $x>0$ and $y>0$, and $f_{X,Y}(x,y)=0$ otherwise. Calculate $P(X+Y\leq 1)$.

4. In the Problem Set #33 from 2014, question #6, we studied an exponential random variable X with $\mathbb{E}(X)=1/3$, and we learned that the distribution of $Y=\lfloor X\rfloor$ is a "Geometric number of losses", i.e., Y is a discrete random variable that has probability mass function $P(Y=y)=q^y p$ for $y\geq 0$, with $p=1-e^{-3}=0.9502$ and $q=e^{-3}=0.0498$.

What is $\mathbb{E}(Y)$? Hint: Since $Y=\lfloor X\rfloor$, then $X\leq Y<X+1$, so $\mathbb{E}(X-1)\leq\mathbb{E}(Y)\leq\mathbb{E}(X)$, i.e., in this case, $-2/3\leq\mathbb{E}(Y)<1/3$. So your answer should be between $-2/3$ and $1/3$.