STAT/MA 41600
In-Class Problem Set #8: September 8, 2017
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Problem Set 8 Answers

1. We have

$P(X=0) = \frac{\binom{3}{0}\binom{6}{5}}{\binom{9}{5}} = \frac{(1)(6)}{126} = 1/21, \qquad P(X=1) = \frac{\binom{3}{1}\binom{6}{4}}{\binom{9}{5}} = \frac{(3)(15)}{126} = 5/14,$

$P(X=2) = \frac{\binom{3}{2}\binom{6}{3}}{\binom{9}{5}} = \frac{(3)(20)}{126} = 10/21, \qquad P(X=3) = \frac{\binom{3}{3}\binom{6}{2}}{\binom{9}{5}} = \frac{(1)(15)}{126} = 5/42.$

Alternatively, we can compute

$P(X=0) = (6/9)(5/8)(4/7)(3/6)(2/5) = \frac{1}{21}$, $P(X=1) = \binom{5}{1}(3/9)(6/8)(5/7)(4/6)(3/5) = \frac{5}{14}$, $P(X=2) = \binom{5}{2}(3/9)(2/8)(6/7)(5/6)(4/5) = \frac{10}{21}$, $P(X=3) = \binom{5}{3}(3/9)(2/8)(1/7)(6/6)(5/5) = \frac{5}{42}$.

2a. We have $P(X>4) = \sum_{x=5}^{\infty}(2/7)(5/7)^{x-1} = (2/7)(5/7)^4\sum_{x=0}^{\infty}(5/7)^x = \frac{(2/7)(5/7)^4}{1-5/7} = (5/7)^4$.

Alternatively, we have $P(X>4) = 1 - P(X=1) - P(X=2) - P(X=3) - P(X=4) = 1 - \sum_{x=1}^{4}(2/7)(5/7)^{x-1} = 1 - (2/7)\frac{1-(5/7)^4}{1-5/7} = (5/7)^4$.

2b. We have $P(X>k) = \sum_{x=k+1}^{\infty}(2/7)(5/7)^{x-1} = (2/7)(5/7)^k\sum_{x=0}^{\infty}(5/7)^x = \frac{(2/7)(5/7)^k}{1-5/7} = (5/7)^k$.

Alternatively, we have $P(X>k) = 1 - P(X=1) - P(X=2)\cdots - P(X=k) = 1 - \sum_{x=1}^{k}(2/7)(5/7)^{x-1} = 1 - (2/7)\frac{1-(5/7)^k}{1-5/7} = (5/7)^k$.

2c. We have $P(3 \le X \le 10) = \sum_{x=3}^{10}(2/7)(5/7)^{x-1} = (2/7)\frac{(5/7)^2-(5/7)^{10}}{1-5/7} = (5/7)^2 - (5/7)^{10}$.

Alternatively, we have $P(3 \le X \le 10) = P(X>2) - P(X>10) = (5/7)^2 - (5/7)^{10}$.

2d. We have $P(i \le X \le j) = \sum_{x=i}^{j}(2/7)(5/7)^{x-1} = (2/7)\frac{(5/7)^{i-1}-(5/7)^j}{1-5/7} = (5/7)^{i-1} - (5/7)^j$.

Alternatively, we have $P(i \le X \le j) = P(X>i-1) - P(X>j) = (5/7)^{i-1} - (5/7)^j$.

3. We have $P(X \ge 7) = P(X=7) + P(X=8) + P(X=9) + \cdots + P(X=18) = 15/216 + 21/216 + 25/216 + 27/216 + 27/216 + 25/216 + 21/216 + 15/216 + 10/216 + 6/216 + 3/216 + 1/216 = 196/216$. Alternatively, we have $P(X \ge 7) = 1 - P(X=3) - P(X=4) - P(X=5) - P(X=6) = 1 - 1/216 - 3/216 - 6/216 - 10/216 = 1 - 20/216 = 196/216$.

4. We have $P(X=0) = \frac{\binom{3}{0}\binom{3}{3}}{\binom{6}{3}} = 1/20$ $P(X=1) = \frac{\binom{3}{1}\binom{3}{2}}{\binom{6}{3}} = 9/20$, $P(X=2) = \frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} = 9/20$, and $P(X=3) = \frac{\binom{3}{3}\binom{3}{0}}{\binom{6}{3}} = 1/20$.