STAT/MA 41600
In-Class Problem Set #11: September 15, 2017
Solutions by Mark Daniel Ward

Problem Set 11 Answers

1. We let X_1 denote the value on the 4-sided die, and X_2 the value on the 6-sided die. So we have $\mathbb{E}(X_1) = (1/4)(1+2+3+4) = 5/2$ and $\mathbb{E}(X_2) = (1/6)(1+2+3+4+5+6) = 7/2$. So we conclude $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2) = \mathbb{E}(X_1) + \mathbb{E}(X_2) = 5/2 + 7/2 = 6$.

2. We let $X_j = 1$ if the jth flip is a head, and $X_j = 0$ otherwise. Therefore, we have $\mathbb{E}(X_j) = (1/2)(1) + (1/2)(0) = 1/2$. We conclude that $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_5) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_5) = 1/2 + \cdots + 1/2 = 5/2$.

3a. We let $X_j = 1$ if the jth card is a Queen, and $X_j = 0$ otherwise. Therefore, we have $\mathbb{E}(X_j) = (4/52)(1) + (48/52)(0) = 1/13$. We conclude that $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_5) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_5) = 1/13 + \cdots + 1/13 = 5/13$.
3b. Same method and same answer as in **3a**.

4. We let $X_j = 1$ if the jth child chosen is a girl, and $X_j = 0$ otherwise. Therefore, we have $\mathbb{E}(X_j) = (3/6)(1) + (3/6)(0) = 1/2$. We conclude that $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 1/2 + 1/2 + 1/2 = 3/2$.