STAT/MA 41600
In-Class Problem Set #25: October 13, 2017

1. Consider a pair of continuous random variables X and Y that have the joint probability density function

$$f_{X,Y}(x,y) = e^{-x-y}, \qquad \text{for positive } x \text{ and } y,$$

and $f_{X,Y}(x,y) = 0$ otherwise. Find the probability that Y is larger than $2X+1$, i.e., calculate $P(Y > 2X+1)$.

2a. For the setup in question 1, find $P(\max(X,Y) \leq 1)$.
2b. For the setup in question 1, find $P(\max(X,Y) \leq 2)$.
2c. For the setup in question 1, if $a > 0$, find $P(\max(X,Y) \leq a)$.
2d. Define a new random variable $Z = \max(X,Y)$. In part c, you essentially calculated the CDF for Z. What is the probability density function for Z?

3. Suppose that the probability density function for X is $f_X(x) = x/9$ for $0 \leq x \leq 3$, and $f_X(x) = 2/3 - x/9$ for $3 \leq x \leq 6$, and $f_X(x) = 0$ otherwise.
3a. Calculate $P(|X-3| < 1/2)$.
3b. Calculate $P(|X-3| > 2)$.

4a. What is the probability density function for the random variable X defined in question 1?
4b. What is the cumulative distribution function for the random variable X defined in question 1?
4c. What is the cumulative distribution function for the random variable X defined in question 3? (You will need to define this in pieces, since the probability density function is defined in pieces.)