STAT/MA 41600
In-Class Problem Set #32 part 2: October 30, 2017

1. At a small town Carnival of Fear, on Halloween night, it is rumored that sometime after midnight, there is always a loud "boom!" followed afterwards by a "scream!" Some probability students decided to investigate this claim. After years of scary measurements, they decided to let X denote the time (in seconds) after midnight when the loud boom occurs, and to let Y denote the time (also in seconds) after midnight when the subsequent scream happens. In particular, we have $X < Y$. The students believe that the joint density of X and Y is $f_{X,Y}(x,y) = \frac{3}{64}e^{-x/4-y/8}$ for x, y positive and $f_{X,Y}(x,y) = 0$ otherwise.
1a. Find the probability density function of X.
1b. Find the conditional probability density function of Y, given that $X = 5$. Hint: This $f_{Y|X}(y|5)$ is nonzero for $y > 5$.
1c. Given $X = 5$, what is the conditional probability that $Y > 12$?

2. If a child eats too much candy while Trick Or Treating on Halloween night, he/she will get a stomach ache. The timing when such a stomach ache starts (after eating too much candy) can be modeled by an exponential distribution, with an average of 3 hours. What is the probability that the stomach ache occurs within the first 30 minutes?

3. Linus and Sally both wait in the pumpkin patch for the Great Pumpkin to arrive... but he never seems to show up, and this year, after midnight, they both start to get tired. The times until they fall asleep are independent exponential random variables (they are independent since they are waiting in different parts of the pumpkin patch without talking to each other). They each have expected time of 20 minutes (after midnight) until they fall asleep. What is the probability that they fall asleep within 5 minutes of each other?

4. An old Purdue Probability legend says that, on Halloween, a mouse appears near the Bell Tower at a time U that is uniformly distributed between 12:00 midnight and 12:10 AM. Also, a black-and-gold colored snake appears at a time X (given in minutes after midnight) that is exponentially distributed, with mean 5 minutes. Assume these times are independent. If the mouse appears first (i.e., if $U < X$), then the snake will find the mouse and will be satisfied. Find $P(U < X)$.