

Problem Set 36 Answers

Throughout this problem set, we use Z to denote a standard normal random variable, i.e., with mean 0 and variance 1.

1. Let X_1, \dots, X_6 and Y_1, Y_2, Y_3 denote the weights of Audrey's and Bruce's rocks (respectively). Then $P(X_1 + \dots + X_6 > Y_1 + Y_2 + Y_3) = P(X_1 + \dots + X_6 - Y_1 - Y_2 - Y_3 > 0) = P\left(\frac{X_1 + \dots + X_6 - Y_1 - Y_2 - Y_3 - (6(10) - 3(21))}{\sqrt{(6)(1.5^2) + (3)(2^2)}} > \frac{0 - (6(10) - 3(21))}{\sqrt{(6)(1.5^2) + (3)(2^2)}}\right) = P(Z > 0.59) = 1 - P(Z \leq 0.59) = 1 - 0.7224 = 0.2776$.

2. We let X_1, \dots, X_{23} denote the quantity of gas purchased by the 23 drivers, so that the total purchase price, $10(X_1 + \dots + X_{23})$ is a Normal random variable with mean $(10)(23)(2.60) = 598$ and variance $(10^2)(23)(0.10^2) = 23$. So we have $P(10(X_1 + \dots + X_{23}) \geq 600) = P\left(\frac{10(X_1 + \dots + X_{23}) - 598}{\sqrt{23}} \geq \frac{600 - 598}{\sqrt{23}}\right) = P(Z \geq 0.42) = 1 - P(Z < 0.42) = 1 - 0.6628 = 0.3372$.

An alternative approach is: $P(10(X_1 + \dots + X_{23}) \geq 600) = P(X_1 + \dots + X_{23} \geq 60) = P\left(\frac{X_1 + \dots + X_{23} - (23)(2.60)}{\sqrt{(23)(0.10^2)}} \geq \frac{60 - (23)(2.60)}{\sqrt{(23)(0.10^2)}}\right) = P(Z \geq 0.42) = 1 - P(Z < 0.42) = 0.3372$.

3. We have $P(X > Y + Z) = P(X - Y - Z > 0) = P\left(\frac{X - Y - Z - (5 - 5 - 5)}{\sqrt{20 + 20 + 20}} > \frac{0 - (5 - 5 - 5)}{\sqrt{20 + 20 + 20}}\right) = P(Z > 0.65) = 1 - P(Z \leq 0.65) = 1 - 0.7422 = 0.2578$.

4ab. In question 4c from the previous problem set, we discovered that the probability that a book is heavy is $p = 0.1446$. So the number of books that she needs to try is a Geometric random variable with $p = 0.1446$, which must have mean $1/p = 6.9156$ and variance $q/p^2 = 40.9103$.

4c. Let X_1, \dots, X_5 denote the weights of the 5 books, in ounces. The probability that the weights of the 5 books exceeds 5 pounds (i.e., 80 ounces) is $P(X_1 + \dots + X_5 > 80) = P\left(\frac{X_1 + \dots + X_5 - (5)(14.2)}{\sqrt{(5)(1.7^2)}} > \frac{80 - (5)(14.2)}{\sqrt{(5)(1.7^2)}}\right) = P(Z > 2.37) = 1 - P(Z \leq 2.37) = 1 - 0.9911 = 0.0089$.