STAT/MA 41600

In-Class Problem Set #39: November 13, 2017

1. As in question 3abc on Problem Set 12, suppose we draw 5 cards at random, without replacement, from a deck of 52 cards (such a deck includes 4 Queens). Let X denote the number of Queens drawn. Define $X_i = 1$ if the ith card selected is a Queen, and $X_i = 0$ otherwise.

1a. Find $\text{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 5$.

1b. Find $\text{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 5$ and $1 \leq j \leq 5$ with $i \neq j$.

1c. Use your work from 1a and 1b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 3c on Problem Set 12?

2. As in question 4 on Problem Set 12, a family with three daughters and three sons needs to go to the grocery store. Besides the father, who is driving the car, exactly three of the children can come along to the grocery store with him. Suppose that the three children to join the father are chosen randomly, and all such choices are equally likely. Let X denote the number of daughters who accompany the father to the grocery. Define $X_i = 1$ if the ith child who joins the father is a girl, and $X_i = 0$ otherwise.

2a. Find $\text{Cov}(X_i, X_i) = \mathbb{E}(X_i X_i) - \mathbb{E}(X_i)\mathbb{E}(X_i)$ for $1 \leq i \leq 3$.

2b. Find $\text{Cov}(X_i, X_j) = \mathbb{E}(X_i X_j) - \mathbb{E}(X_i)\mathbb{E}(X_j)$ for $1 \leq i \leq 3$ and $1 \leq j \leq 3$ with $i \neq j$.

2c. Use your work from 2a and 2b to obtain the variance of X (i.e., the covariance of X with itself). Does it agree with the solution to 4c on Problem Set 12?

3. Consider a pair of random variables X and Y whose joint probability density function is constant on the triangle with vertices at the points $(-4, 0)$, $(0, 2)$, and $(8, 0)$. Setup the integral $\text{Cov}(X, Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x - 4/3)(y - 2/3)f_{X,Y}(x, y)\,dx\,dy$ and calculate it directly. (We are using the fact, as in Problem Set 28, that $\mathbb{E}(X) = 4/3$ and $\mathbb{E}(Y) = 2/3$.)

After you compute the double integral, make sure your solution agrees with $\text{Cov}(X, Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) = 2/3 - (4/3)(2/3) = -2/9$. (Here we are using the results from Problem Set 29, question 4a.)

4. As in question 1 on Problem Set 26, suppose that X and Y have joint probability density function $f_{X,Y}(x, y) = (3/4)(x - y)$ for $0 < y < x < 2$, and $f_{X,Y}(x, y) = 0$ otherwise. Find $\text{Cov}(X, Y)$.